Intensity-Duration Relation in the Bartlett-Lewis Rectangular Pulse Model

Christoph Ritschel(1), Henning W. Rust(1), Peter Névir(1) and Uwe Ulbrich(1)

(1) Institut für Meteorologie, Freie Universität Berlin, Germany

Freie Universität Berlin

Introduction

High resolution (sub-daily) precipitation time series are indispensable for hydrological applications but not always available. As a replacement simulations with stochastic precipitation models are used (e.g., Onof. et al., 2000) A canonical class of sub-daily stochastic precipitation models are Poisson cluster models, e.g., the Bartlett-Lewis rectangular pulse model (BLRPM) (e.g., Wheater et al., 2005). The BLRPM has been shown to be able to well reproduce certain characteristics found in observations (Rodriguez-Iturbe et al., 1987).
As intensity-duration-frequency relationships are of particular interest to hydrological impact assessment, we address the following question for a case study: **Is the BLRPM able to reproduce the intensity-duration relationship found in observations?**

Intensity-Duration-Frequency (IDF)

Intensity-duration-frequency (IDF) curves show *return levels* as a function of *rainfall duration*, typically for a range of *return periods*. We estimate IDF curves from monthly maxima of precipitation with different aggregation times (1h to 72h) using the generalised extreme value (GEV) distribution (Coles et al., 2001) and the following relation for different durations d Koutsoyiannis et al. (1998):

- $\tilde{\mu} = \mu/\sigma_d$ is independent of the durations d
- ξ is independent of the durations d
- σ_d can be modelled as a function of duration d, Eq. (1)

$$\sigma_d = \frac{\sigma}{(d+\theta)^\eta} \qquad (1)$$

with θ, η and σ being independent of d.

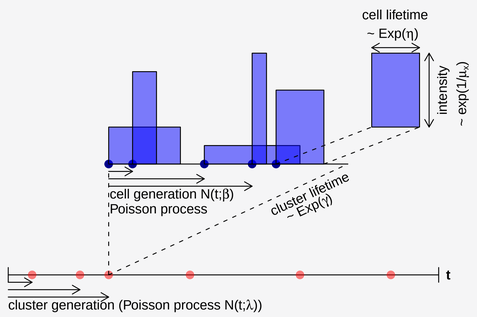
This lead to the following formulation for a duration dependent GEV

$$F(x;\mu,\sigma_d,\xi) = \exp\left\{-\left[1+\xi\left(\frac{x}{\sigma_d}-\mu\right)\right]^{\frac{-1}{\xi}}\right\} \qquad (2)$$

which allows simultaneous modelling of rainfall maxima across different durations using a single GEV distribution at the cost of only two additional parameters which can be estimated by maximum-likelihood (Soltyk et al., 2014).

Bartlett-Lewis rectangular pulse model

The BLRPM is a stochastic precipitation model with two levels of Poisson-processes (Rodriguez-Iturbe et al., 1987) and a set of five physically meaningful parameters: $\boldsymbol{\theta} = \{\lambda, \gamma, \beta, \eta, \mu_x\}$



- λ: expected number of clusters per simulation time
- γ: inverse of expected cluster lifetime
- β: expected number of cells per cluster lifetime
- η: inverse of expected cell lifetime
- μ_x: expected cell intensity

Figure 1 : Scheme of the Bartlett-Lewis rectangular pulse model, similar to Wheater et al. (2005)

BLRPM parameters can be estimated by a method of moments:

- define a set (e.g., $k = 13$) of precip. characteristics T_i, e.g. mean or variance for different aggregation times
- obtain these characteristics from the model parameters $\rightarrow \tau_i(\boldsymbol{\theta})$ and
- equate to those from observations (T_i) and minimise a weighted objective function

$$\mathrm{OF}(\boldsymbol{\theta};\mathrm{T}) = \sum_{i=1}^{k} w_i \left\{\left[1-\frac{\tau_i(\boldsymbol{\theta})}{T_i}\right]^2 + \left[1-\frac{T_i}{\tau_i(\boldsymbol{\theta})}\right]^2\right\} \text{ with } w_{i=1} = 100 \text{ and } w_{i\neq 1} = 1 \qquad (3)$$

Data

A high resolution (1-min) precipitation time series for Berlin-Dahlem (2001-2013), aggregated for 1, 2, 3, 6, 12, 24, 48 and 72 hours. BLRPM parameters are estimated based on 1h mean and 1, 3, 12 and 24 hour variance, lag-1 auto-covariance and probability of zero rainfall. IDF parameters are estimated using annual maxima for each month for the set of durations as input.

Seasonality of BLRPM parameters

Figure 2 depicts the seasonality of BLRPM parameters μ_x (cell intensity) and η (expected number of cells).

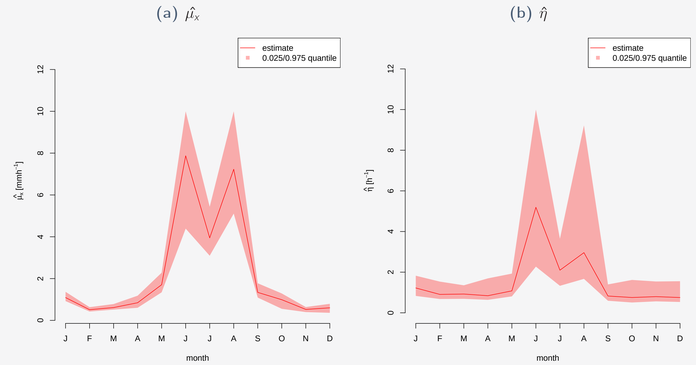


Figure 2 : Seasonality of estimated BLRPM model parameters a) $\hat{\mu}_x$ and b) $\hat{\eta}$. Shown are the estimated parameters (red) with confidence intervals (0.025/0.975 quantile, blue) from a parametric bootstrap for Berlin-Dahlem.

- large intensities (μ_x) and short storm durations (large η) in summer correspond to precipitation being dominated by convective events

Seasonality of IDF parameters

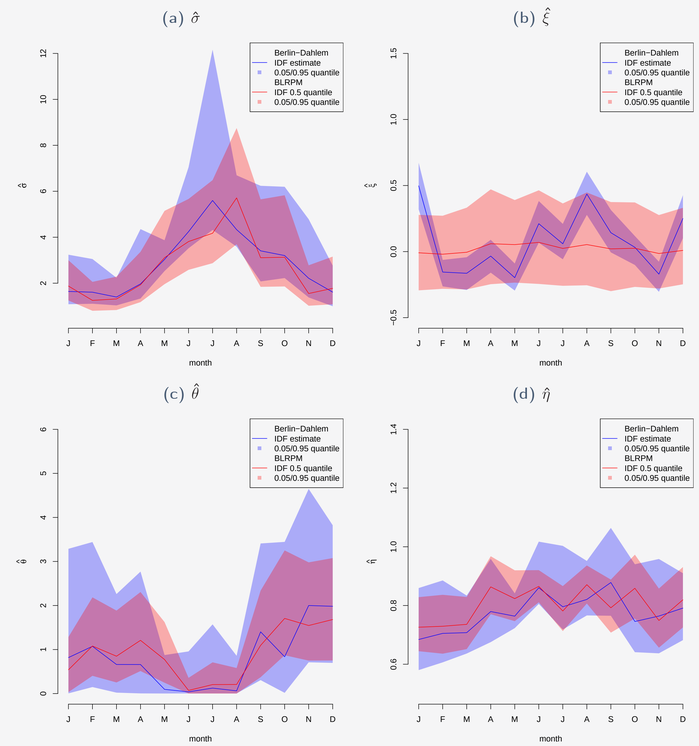


Figure 3 : Seasonality of IDF model parameters estimated for Berlin-Dahlem (blue line) with confidence intervals (0.025/0.975) from a parametric bootstrap (blue layer). In addition the 0.5 quantile (red line) and the 0.025/0.975 quantiles (red layer) of 1000 BLRPM simulations are shown.

- The BLRPM can reproduce the seasonality in IDF parameters from observations (Fig. 3); $\hat{\sigma}$, $\hat{\theta}$ and $\hat{\eta}$ are well reproduced; more difficult are shape $\hat{\xi}$ and the location $\hat{\mu}$ (not shown)
- large scales ($\hat{\sigma}$), small curvature ($\hat{\theta}$) and steep slopes ($\hat{\eta}$) in the double-logarithmic IDF are consistent with the dominant convective events in summer, compare also Fig. 4

IDF curves

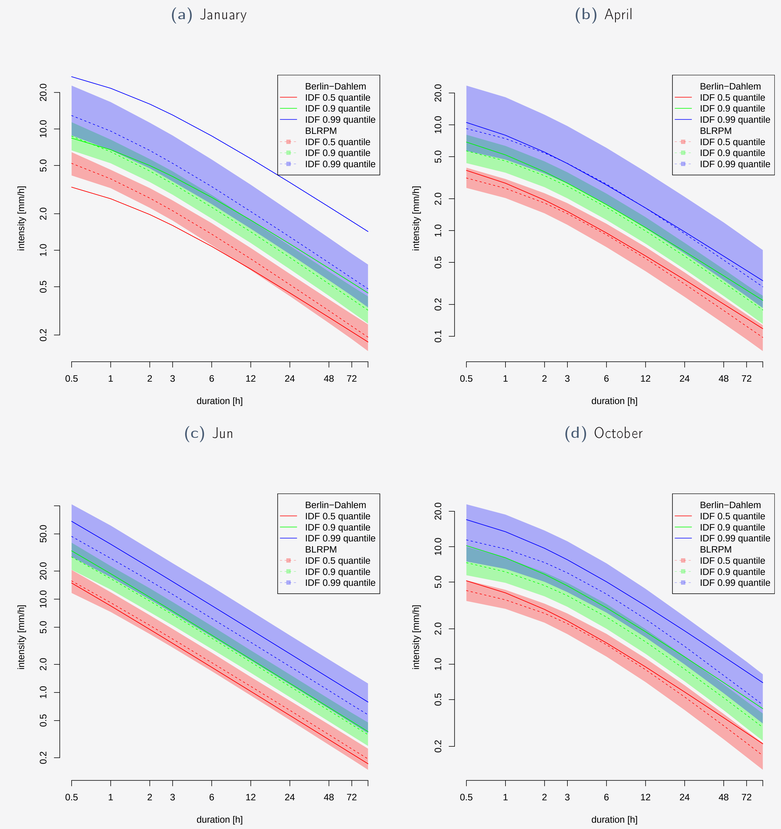


Figure 4 : IDF curves for a) January, b) April, c) June and d) October. Shown are the 0.5 (red), 0.9 (green) and 0.99 (blue) quantile from observations at Berlin-Dahlem (solid lines). The dashed lines and surrounding shadows are the respective 0.5, 0.025 and 0.975 quantiles for the associated IDF curves obtained from 1000 simulations of a BLRPM with parameters estimated for Berlin-Dahlem.

- IDF curves for Berlin-Dahlem can be well reproduced with BLRPM simulated rainfall across the year
- discrepancies for January are likely due to Kyrill (18/19.01.2007) with extreme precipitation which lead to a large shape parameter; the outstanding value does not propagate to the BLRPM parameters and lead thus to smaller spacings between the quantiles in the IDF plot

Summary and Outlook

The BLRPM is in general able to reproduce the IDF characteristics of Berlin-Dahlem precipitation observations.

- BLRPM and IDF parameters can be related to local precipitation characteristics (e.g., dominant convective events in summer)
- outstanding extremes (e.g. Kyrill) are problematic in a relatively short series

Outlook

- investigate IDF curves for modified versions of the BLRPM
- can we exploit the intensity-duration relation to reduce parameters in the BLRPM?

Literature

S. Coles, J. Bawa, L. Trenner, and P. Dorazio. *An introduction to statistical modeling of extreme values*, volume 208. Springer, 2001.

D. Koutsoyiannis, D. Kozonis, and A. Manetas. A mathematical framework for studying rainfall intensity-duration-frequency relationships. *Journal of Hydrology*, 206(1):118–135, 1998.

C. Onof., R.E. Chandler, A. Kakou, P. Northrop, H.S. Wheater, and V. Isham. Rainfall modelling using poisson-cluster processes: a review of developments. *Stochastic Environmental Research and Risk Assessment*, 14:184–411, 2000.

I. Rodriguez-Iturbe, D.R. Cox, F.R.S., and V. Isham. Some models for rainfall based on stochastic point processes. *Proc. R. Soc. Lond.*, A 410:269–288, 1987.

S. Soltyk, M. Leonard, A. Phatak, E. Lehmann, et al. Statistical modelling of rainfall intensity-frequency-duration curves using regional frequency analysis and bayesian hierarchical modelling. 2014.

H.S. Wheater, R.E. Chandler, C.J. Onof, V.S. Isham, E. Bellone, C. Yang, D. Lekkas, G. Lourmas, and M.-L. Segond. Spatial-temporal rainfall modemodel for flood risk estimation. *Stoch. Environ. Res. Risk. Assess*, 19:403–416, 2005.

http://www.geo.fu-berlin.de/met/ag/clidia/Mitarbeiter/ChristophRitschel

christoph.ritschel@met.fu-berlin.de