NUMERICALLY STABLE APPROACH FOR HIGH-PRECISION ORBIT INTEGRATION USING ENCKE'S METHOD AND EQUINOCTIAL ELEMENTS

MATTHIAS ELLMER AND TORSTEN MAYER-GÜRR
INSTITUTE OF GEODESY, NAWI GRAZ, GRAZ UNIVERSITY OF TECHNOLOGY

Introduction

Dynamic orbits play an important role in the setup of the observation equations in low-low satellite-to-satellite gravity field determination. These orbits are determined through integration of the accelerations acting on a satellite, which can then be added to a known or estimated initial state.

We show investigations into the precision of an improved Enke approach[1] to the numerical integration of dynamic orbits.

Our approach allows for computation of dynamic orbits with repeatability at machine precision over a large swath of the spectral domain.

Methods

We compute 24h dynamic orbit arcs from real data by integrating all acting accelerations (as measured by the accelerometer and computed from gravitational background models) using a polynomial integration approach. An initial orbit is used as a taylor point for the evaluation of force models.

The integrated orbit is then fitted to GPS observations. We use this fitted orbit as the taylor point while repeating the integration. After some iterations, the orbit will converge. This can be observed in the coordinate changes between iterations.

After such convergence occurs, we can compare the results from two succesive iterations of orbit integration. For different integration algorithms, this coordinate difference can be of vastly different magnitude, giving an indication to the performance of the method.

Thus, the magnitude of the orbit difference between iterations after convergence can be used as an indicator for the quality of the integration algorithm.

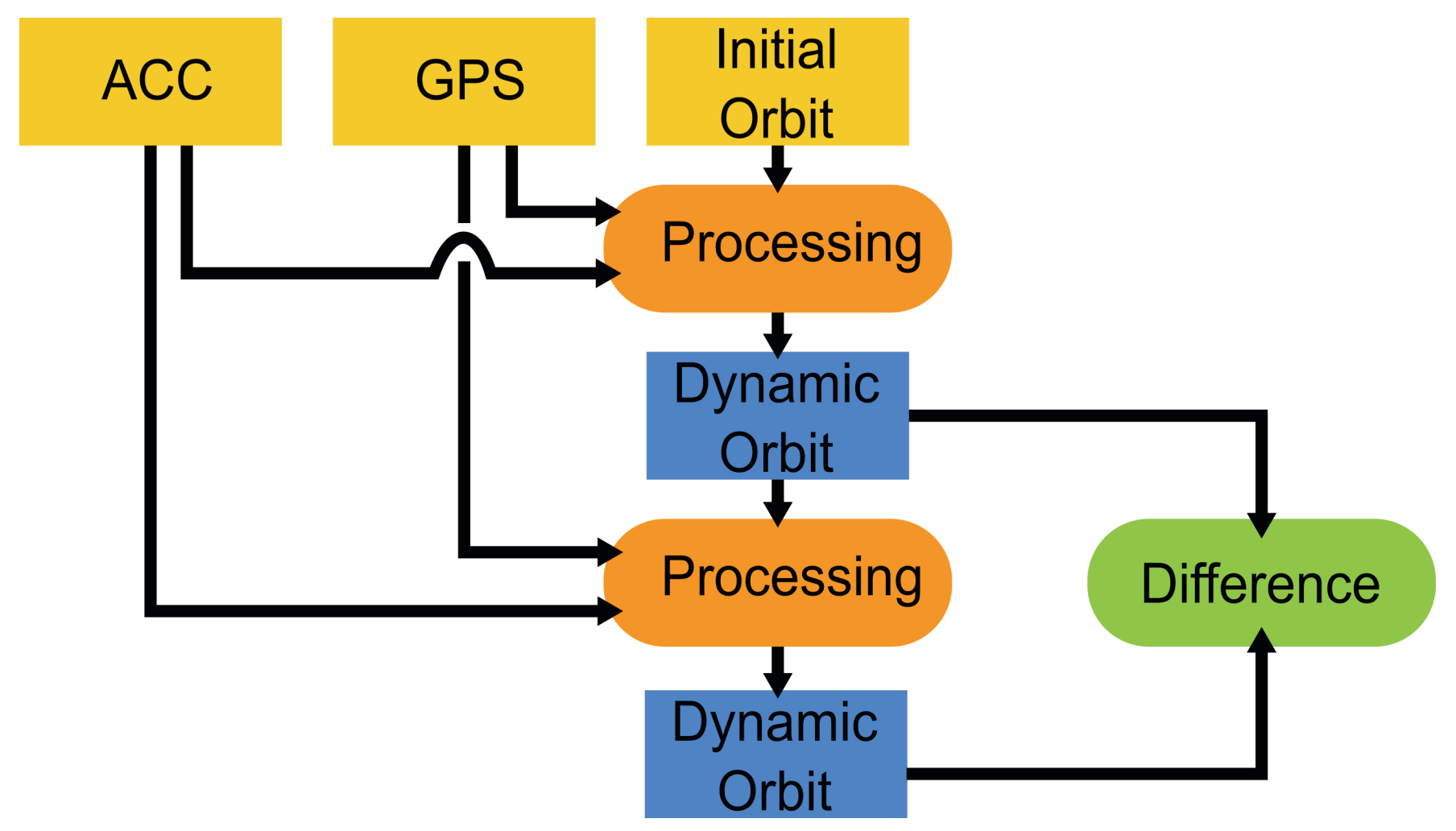
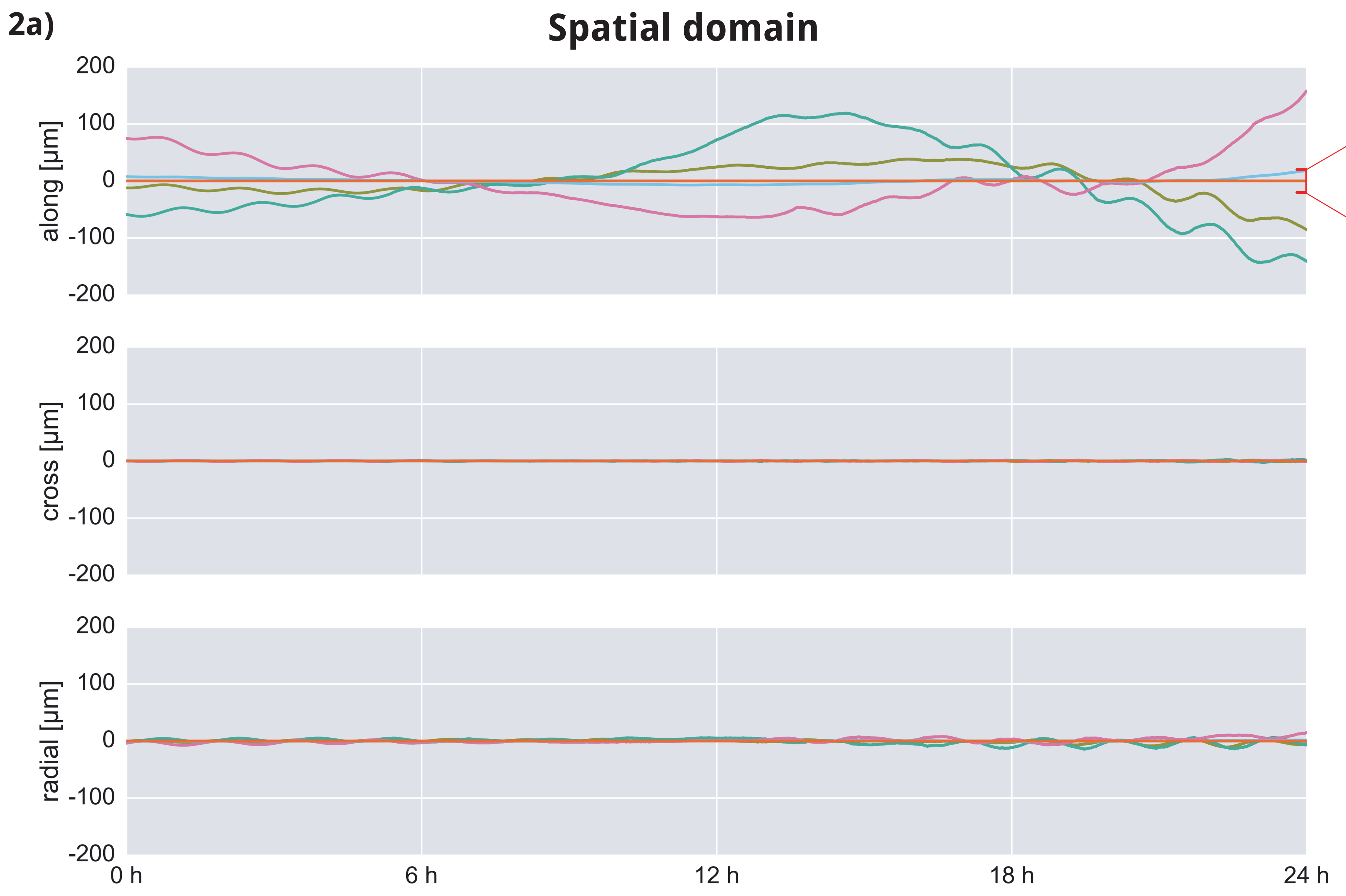
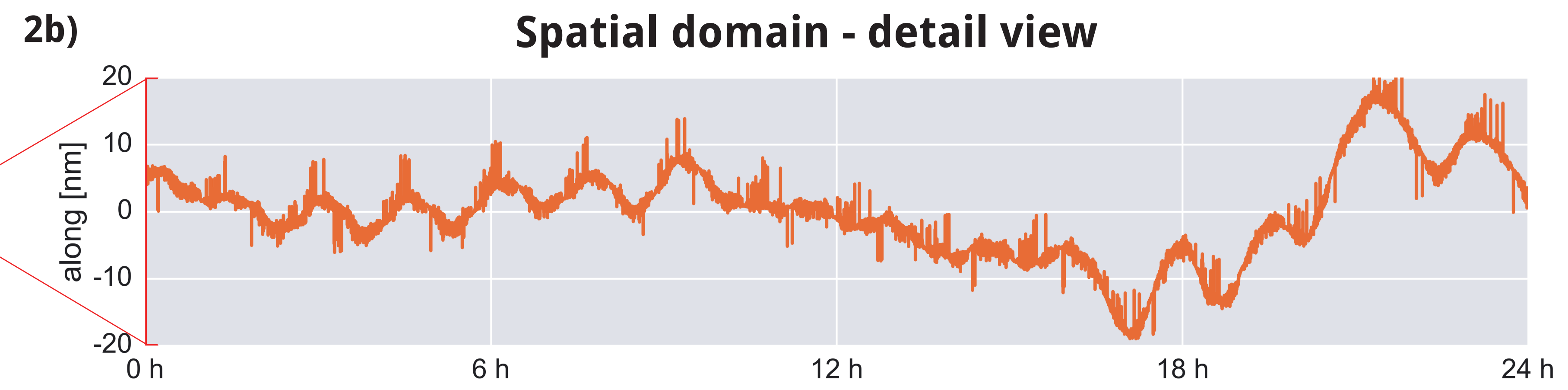


Fig 1: Processing steps from initial observations to comparison result.

Difference between dynamic orbit iterations after convergence





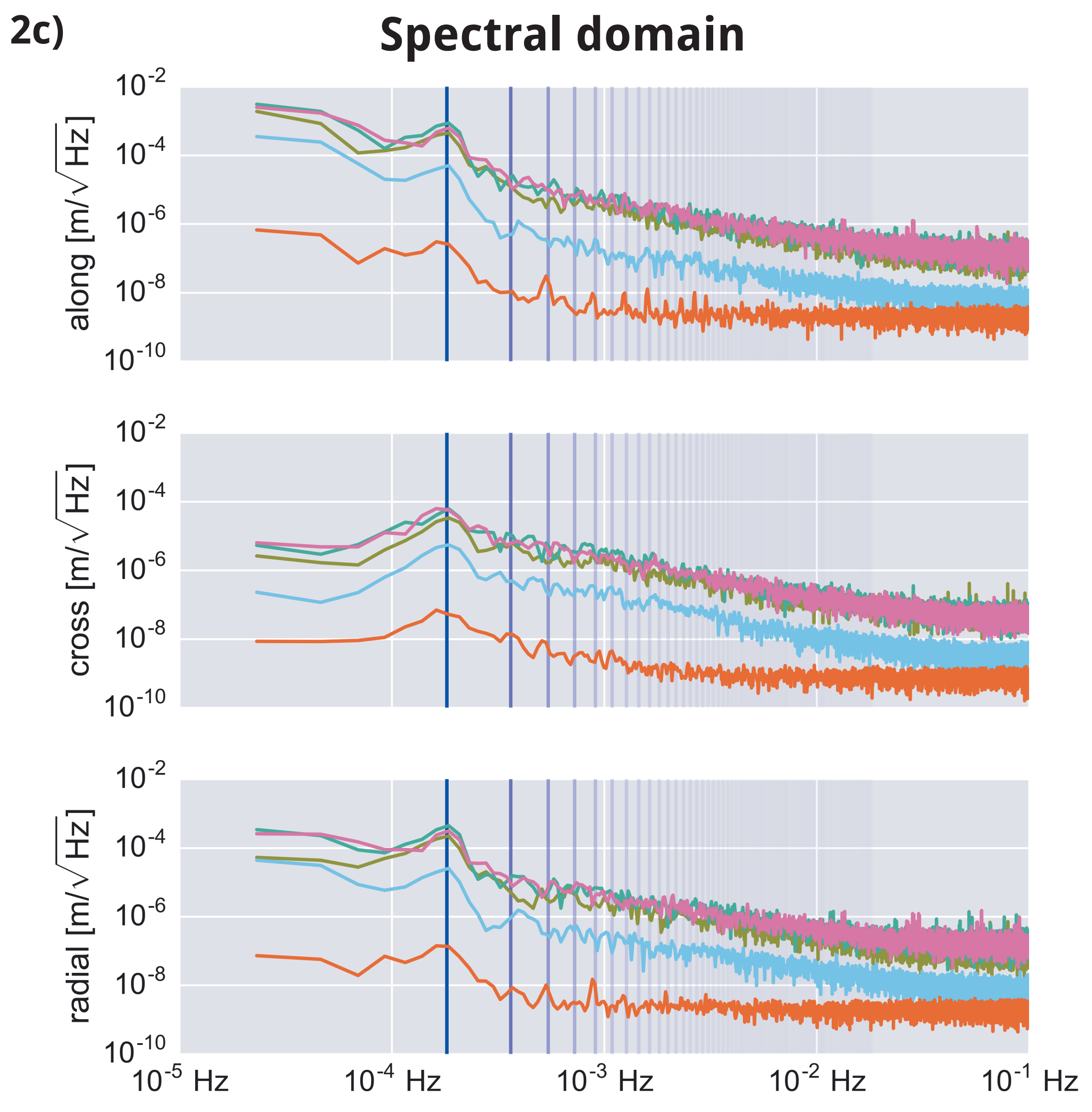


Fig 2a: Shows the difference between orbit iterations in the spatial domain. All integration methods have the largest errors in the along-track axis.

Fig 2b: Detail view of improved Enke method using equinoctial elements. Numerical artefacts due to machine precision visible.

Fig 2c: Shows the difference between orbit iterations in the spectral domain. All integration methods have the largest error at a frequency of once per revolution (dark blue line). Flat parts of the spectrum indicate that the errors in this band are due to machine precision.

Type of reference motion and parametrization

- 🟩 Initial position and velocity
- 🟩 Kepler elements (first epoch)
- 🟪 Kepler elements (best fit)
- 🟦 Equinoctial elements (first epoch)
- 🟧 Equinoctial elements (best fit)

Using a linear reference motion, we observe differences on the order of 100µm between successive integrations. This is magnitudes larger than for example the GRACE K-Band ranging accuracy. As in all other cases, the differences are largest in the along-track component.

Using a first epoch reference ellipse computed with Kepler elements, we can observe no improvement to the integration results over the linear reference motion (see figure 2a). The quality gain from computing a smaller integral is offset by the insufficient accuracy of the reference motion.

Minimizing the forces to be integrated by using a best-fit Kepler ellipse does not lead to better results. The reference motion computed from Kepler elements has insufficient accuracy when comuted in double precision arithmetic.

Going back to a a reference ellipses at the first epoch, use of equinoctial elements for the parametrization leads to significantly smaller deviations between iteration steps, on the order of 20µm (see figure 2a). The overall error in integration is improved by an order of magnitude (see figure 2c).

By using a best-fit reference ellipse, we minimize the power of the computed integral. This leads to a deviation between iterations of only machine precision over a large part of the spectrum (see figure 2c and box Precision). Most of the remaining error is at very long wavelengths, above ~1/rev.

Enke approach to integration

The position of a satellite along its orbit can be interpreted as the sum of a well-described reference motion and the integral of all acting residual accelerations f not inluded in the reference motion.

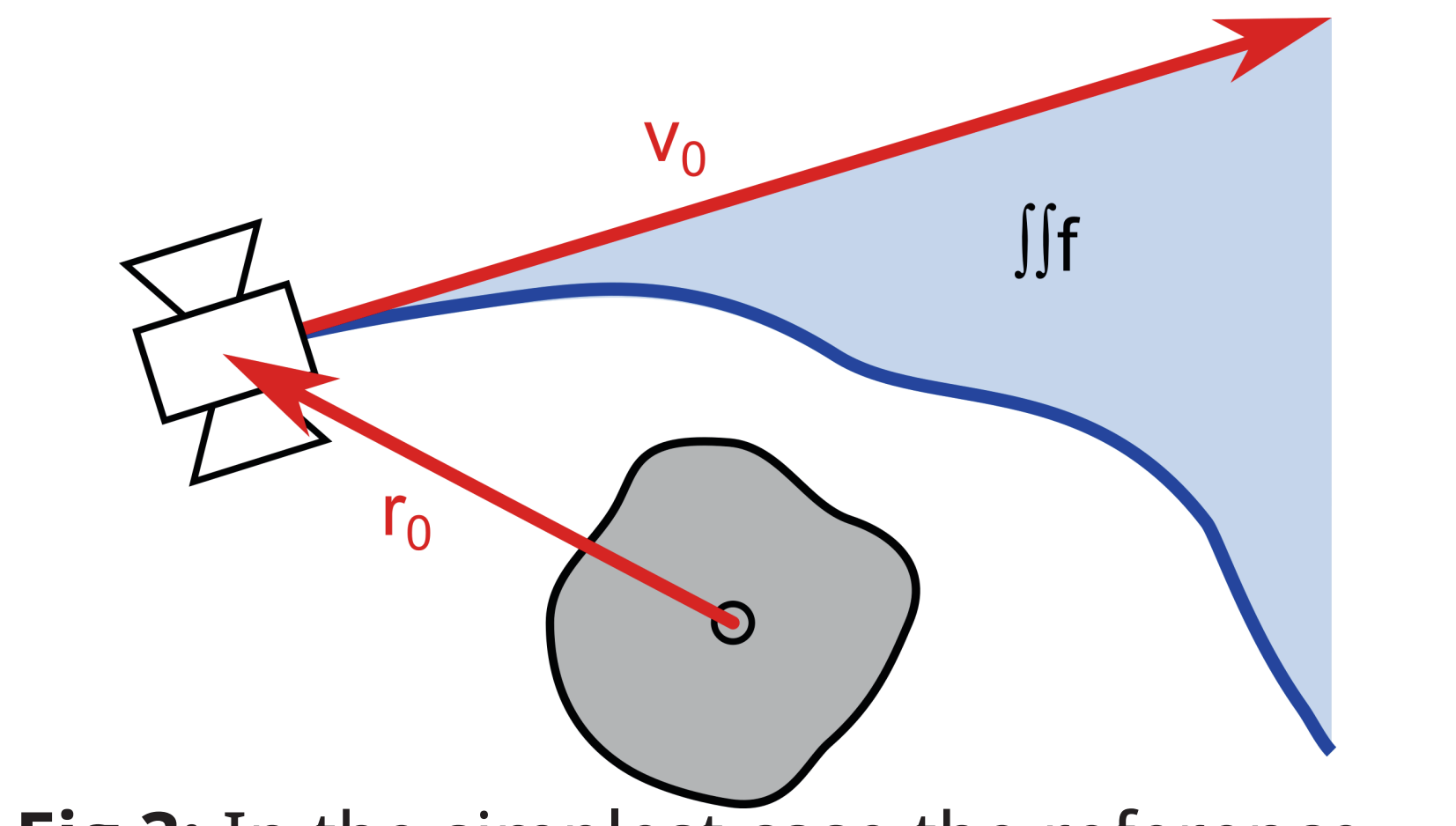


Fig 3: In the simplest case the reference motion is linear, as described by an initial position and velocity r_0, v_0. This may lead to the integrated accelerations f becoming large, and possibly numerically difficult.

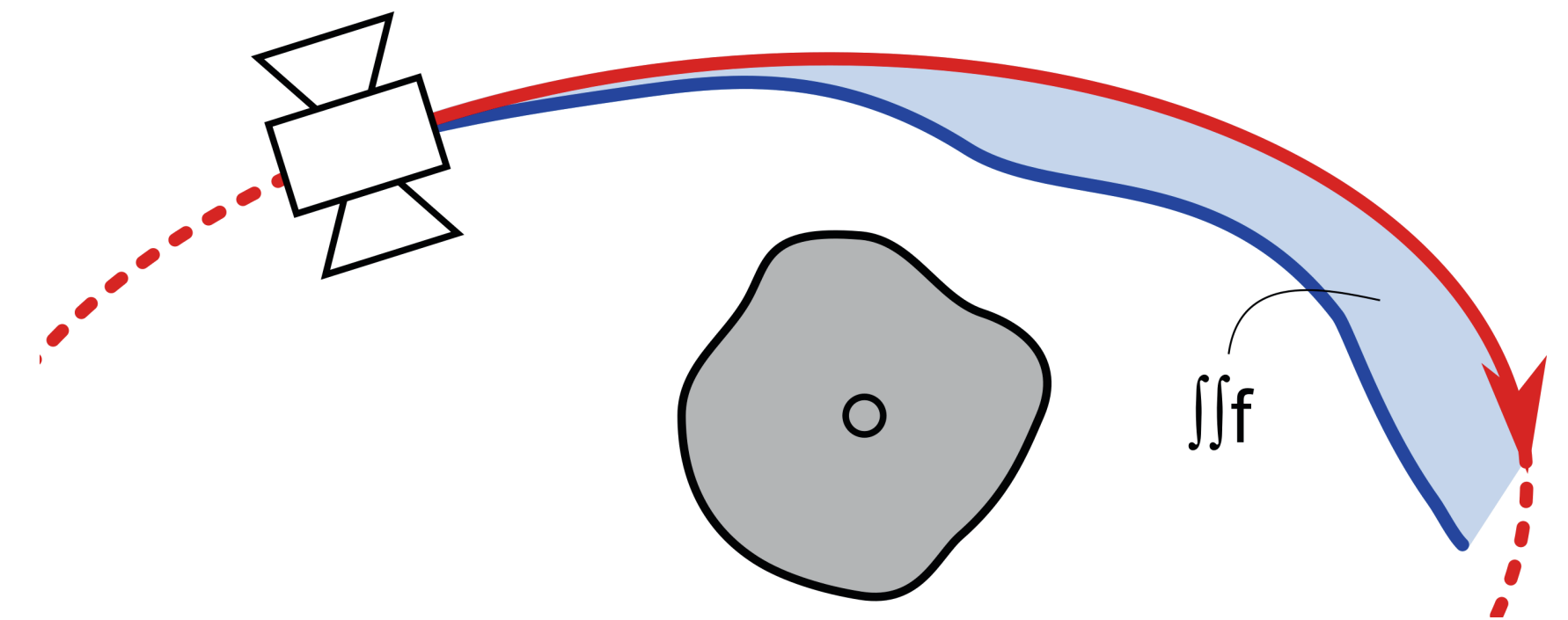


Fig 4: Enke suggests the use of the keplerian ellipse at the first epoch as a reference motion, leading to a smaller integral.

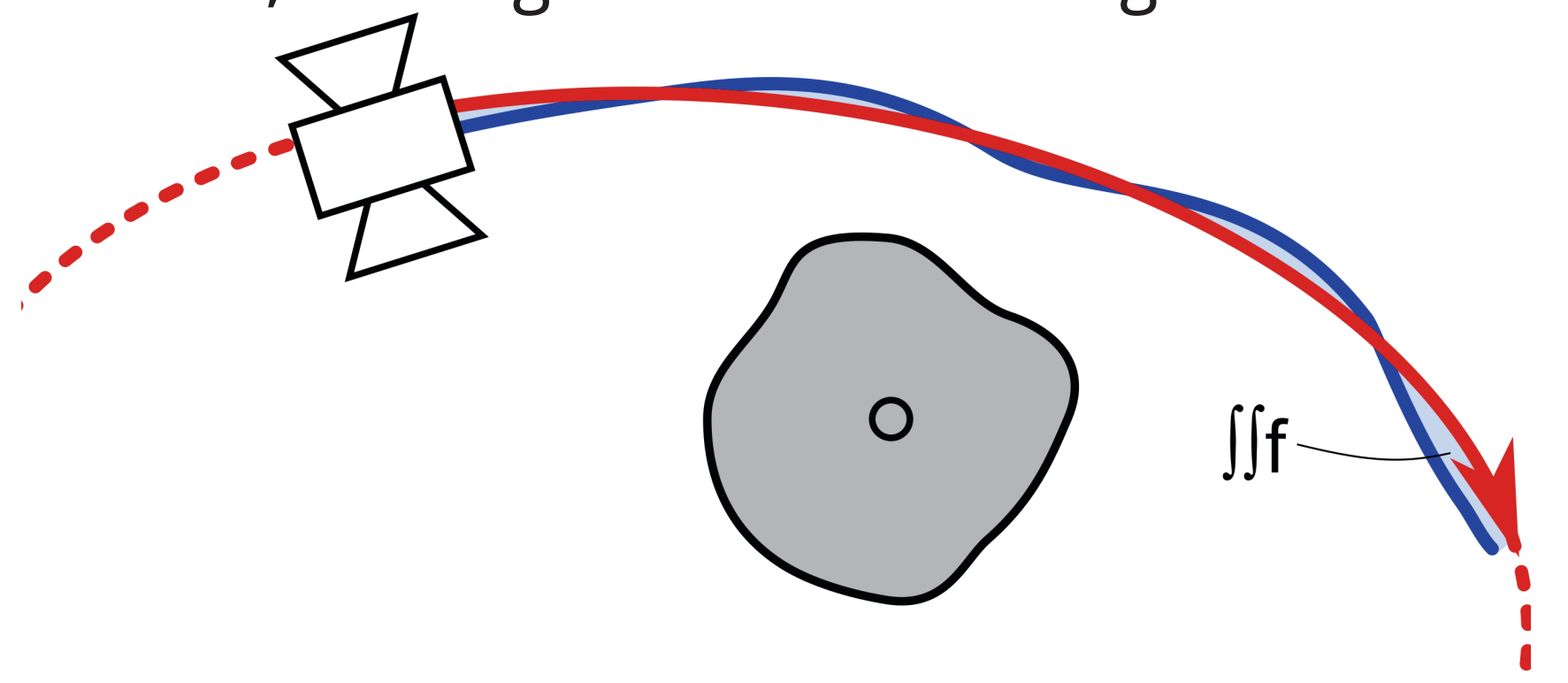


Fig 5: We refine this approach by determining a best-fit orbital ellipse, thus minimizing the energy of the integral of the accelerations.

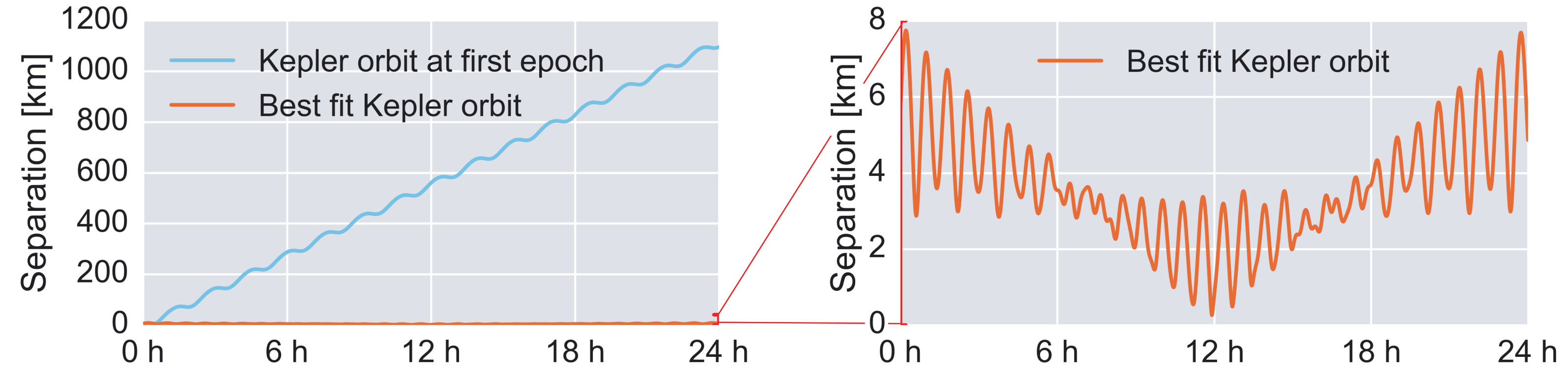


Fig 6: Separation between reference motion and integrated orbit over one day.

Results

We improved on Enke's method by using a best-fit Kepler ellipses as reference motion for dynamic orbit integration.

We show that using equinoctial elements for the parametrization of this ellipse leads to a substantial increase in precision for the result of the dynamic orbit integration.

A need for higher precision would necessitate the consistent use of quadruple precision arithmetic.

Equinoctial elements

The equinoctial elements[2] are non-singular for all eliptical orbits. Position and velocity can be derived from the equinoctial elements with high precision and efficiency, as no trigonometric functions are used. In terms of Kepler elements, the equinoctial elements are given by:

$a = a$
$\lambda = M + \omega + \Omega$
$h = e \sin(\omega+\Omega)$
$k = e \cos(\omega+\Omega)$
$p = \tan(i/2) \sin \Omega$
$q = \tan(i/2) \cos \Omega$

Precision

We inspect the values for one coordinate at a random point along the orbit in two succesive iteration steps:

Linear motion:	Best fit using equinoctials:
6436944.405579335 1m	6436944.4056150075m
6436944.4055785714m	6436944.4056150084m

The improved Enke approach using a best fit Kepler ellipses provides 15 digits of precision.

[1] Encke, Johann Franz. "Über eine neue Methode der Berechnung der Planetenstörungen." Astronomische Nachrichten 33, no. 26 (February 1852): 377–398.
[2] Broucke, R. A., and P. J. Cefola. "On the Equinoctial Orbit Elements." Celestial Mechanics 5, no. 3 (May 1, 1972): 303–310.

MATTHIAS ELLMER
email: ellmer@tugraz.at
phone: +43 316 873 6347
TORSTEN MAYER-GÜRR
email: mayer-guerr@tugraz.at
phone: +43 316 873 6359
itsg.tugraz.at