Imaging density in the Earth and the construction of optimal observables

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Questions?

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Imaging density – the problem

*Density plays a major role in determining the forcings on plate tectonics and mantle convection, but it remains difficult to constrain independently. With the advent of high quality data, powerful computing resources and waveform tomography techniques, however, it is becoming possible to investigate how to image density. This work is described fully in **Blom et al, GJI 2017**.*

S-wave going past a density anomaly

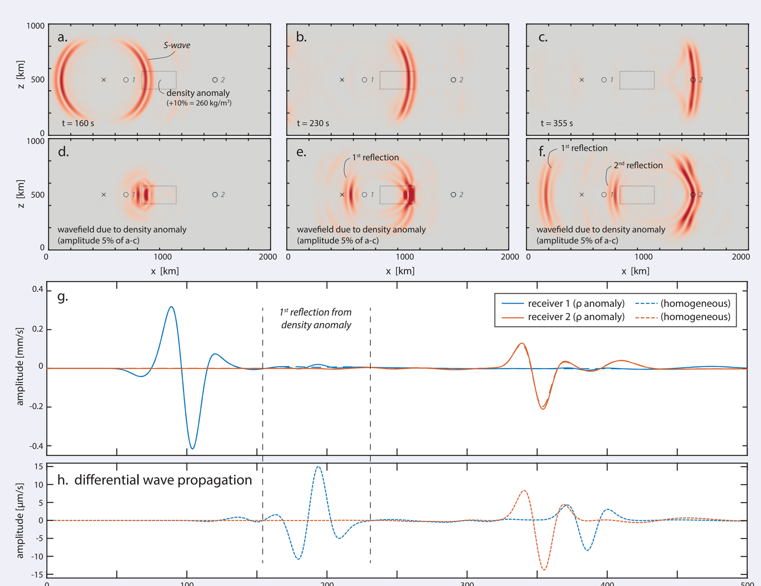

Figure 1. (a–c) Velocity wavefield past a +10% density anomaly.

(d-f) Differential wavefield caused by the anomaly only – amplitudes here are 5 per cent of the amplitudes shown in (a-c).

(g) Seismograms recorded at receivers 1 and 2 for both cases, with and without a density anomaly. At receiver 1, a clear separate arrival is visible caused by the reflection at the first density interface.

(h) Differential seismograms from receivers 1 and 2, obtained by subtracting $v_{diff} = v_{p,anomaly} - v_{homog}$.

1. Synthetic inversion setup & strategy

We perform synthetic waveform tomography experiments in 2-D using the adjoint method in a mantle-sized model (Fig. 2).

- whole mantle setup
- 8 point force sources at (×) 56 km depth
- 16 receivers (o) at the surface
- absorbing boundaries left and right
- bottom boundary reflecting (à la core-mantle boundary)

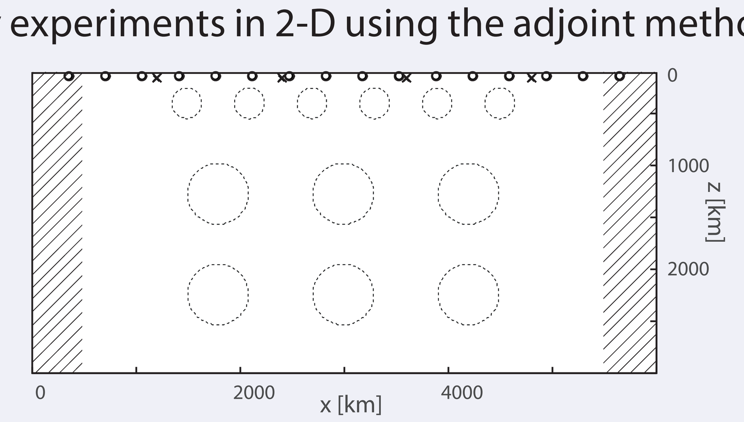

Figure 2. The model layout

The target model is known (Fig. 3). In this model, density, S-velocity and P-velocity are uncorrelated by design. This is because we want to **image density independently** without any prior constraints about its geometry and distribution. **We investigate the following questions**:

- *Can density be imaged as a separate, independent parameter?*
- *What is the effect of ignoring density when density structure is present?*
- *What is the effect of (erroneously) scaling density to S velocity?*
- *Can density be imaged in the presence of noise?*

Synthetic inversion setup

- L2 waveform misfit functional
- L-BFGS optimisation algorithm
- 160 iterations in 8 frequency bands:
  150 – 150 s
  150 – 120 s
  150 – 96 s
  150 – 77 s
  150 – 61 s
  150 – 49 s
  150 – 39 s
  150 – 30 s

References: **Blom, Boehm, Fichtner**, Synthetic inversions for density using seismic and gravity data. Geophys. J. Int (2017) doi: 10.1093/gji/ggx076 **Bernauer et al,** Optimal observables for multiparameter seismic tomography, Geophys. J. Int (2014) doi: 10.1093/gji/ggu204

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2. Synthetic results

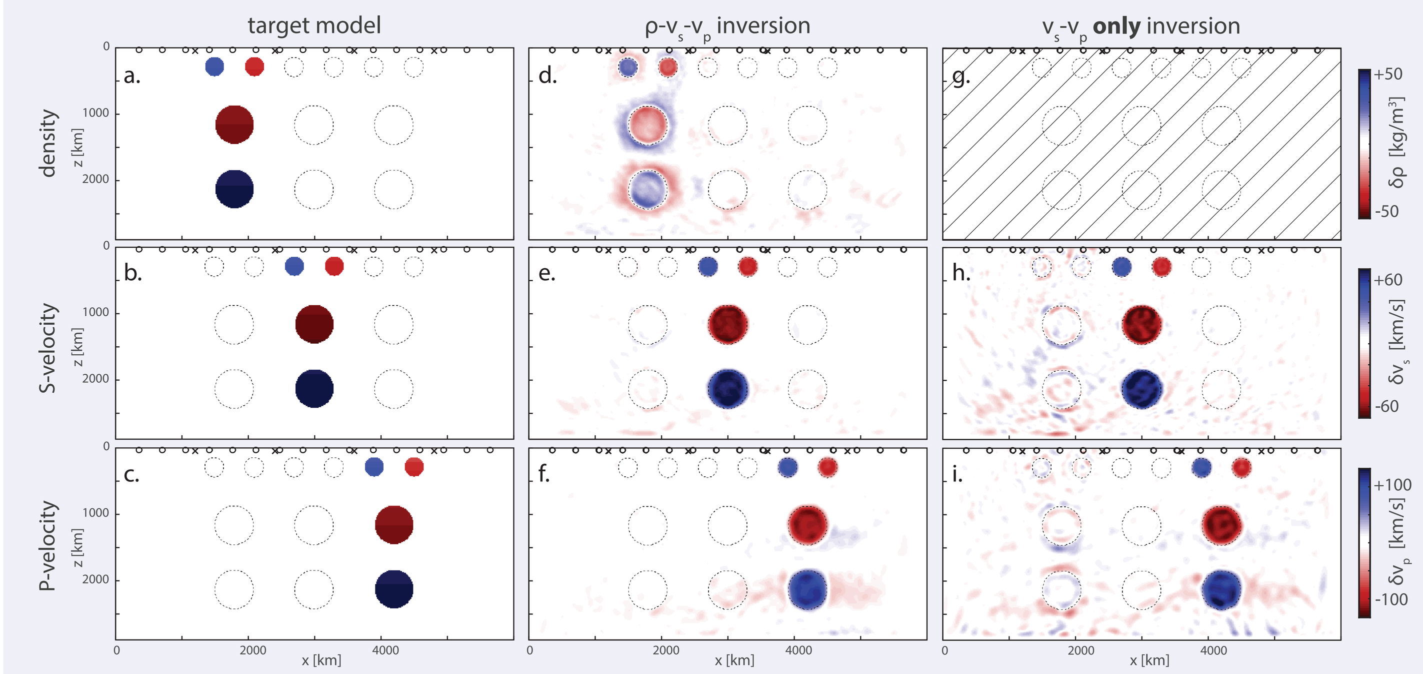


Figure 3. (a-c) **Target model.**

(d-f) Recovered model after 160 iterations when all three parameters density, S velocity and P velocity are free. **Density is best recovered at the edges of the anomalies.**

(g-i) Recovered model for the inversion where only S and P velocities are unconstrained. Density remains fixed at PREM values. The **missing density structure maps into the other parameters** as circular anomalies, mainly at the edges of the locations of the actual density anomalies.

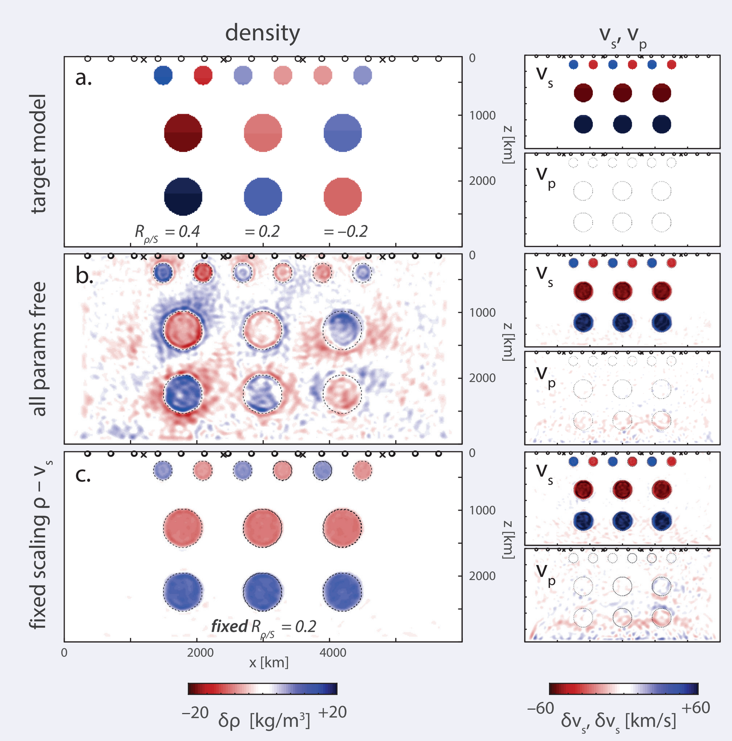


Figure 4. The effect of **imposing a fixed scaling $R_{\rho/S}$** between density anomalies and S-velocity anomalies. Models in density are shown in the large panels, while the small panels show S and P velocity.

(a) Target model in density, where each column is scaled to S velocity (small panel) according to a different scaling - each of these within a reasonable range of Earth-like values.

(b) the recovered density model when **all parameters are free** (like in the reference case above). Here, density is best recovered where it is strongest, and because of the overlying structures, strong artefacts are present all throughout the model domain.

(c) Recovered density model if it is scaled to S velocity with a **fixed $R_{\rho/S}$** = 0.2. The middle column is here correct, and there are much fewer artefacts. However, all the interesting information on the two other columns, whose scaling deviates from the imposed value, is completely lost. In this case, more artefacts are present in the recovered P model.

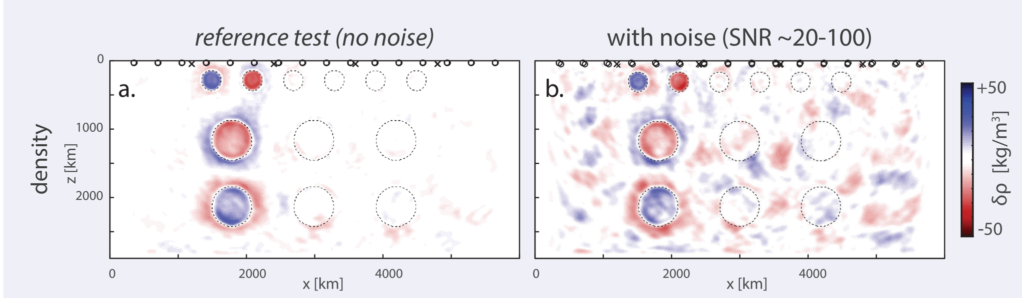


Figure 5. (a) Reference test. (b) Density model in an inversion where all data traces had noise added to them - around 5% (SNR 20) at the lowest frequencies, decreasing to ~1% (SNR 100) for the highest frequencies.

3. Towards Optimal Observables

*Despite the fact that density can clearly be recovered (box 2), the density effect on waveforms remains weak (Fig 1h), and trade-offs persist (e.g. Fig 4d). The method of Optimal Observables as developed by **Bernauer et al (GJI 2014)** is excellently suited to address in particular the issue of trade-offs.*

4. Theory of Optimal Observables

We define a joint optimisation problem in which we try to maximise sensitivity to one parameter (expressed as sensitivity power) whilst minimising sensitivity to the others.

$$\max(P) \quad \text{with} \quad P = \sum_p P_p \quad \text{and} \quad P_p = b_p \int_V {K_p}^2(\mathbf{x})\, d\mathbf{x} \quad \text{and} \quad K_p = \sum_o w_o K_{o,p}$$

with $b > 0$ for $p = \rho$ and $b < 0$ for $p = v_{sh}, v_{sv}, v_p, \dots$. o are the basic observables.

Solving this will result in the vector of observable weights ***w*** which gives optimal observables with sensitivity maximised for the wanted parameter, and minimised for the others. A number of (subjective) decisions determine the shape of the problem and outcome:

- the choice of **basic observables**.
- the question is how to weigh the different parts (rather maximise sensitivity to density, or minimise sensitivity to the others?). This is expressed in the **weighting vector *b*** and the **optimality criterion.**

4. Toy problems with two observables

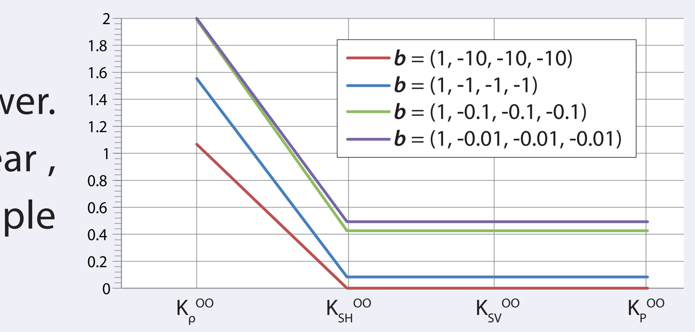
Scenario 1: For any ***b***, sensitivity to density is optimised when observable 1 gets full weight and 2 gets none. This is because ρ is linearly independent from the other parameters.

	K_ρ	K_{SH}	K_{SV}	K_p
obs.1	1	0	0	0
obs.2	0	1	1	1

Scenario 2: Now the observable weights depend on the choice of ***b***. As in realistic applications, parameters are not linearly independent - resulting in variable optimal observables..

	K_ρ	K_{SH}	K_{SV}	K_p
obs.1	1	0	0	0
obs.2	1	1	1	1

The most expensive step is calculating the kernel-kernel products required for the sensitivity power. Once this is done, finding the optimal ***b*** (a non-linear, but rather cheap problem) can be done with a simple grid search.



6. Eastern Mediterranean

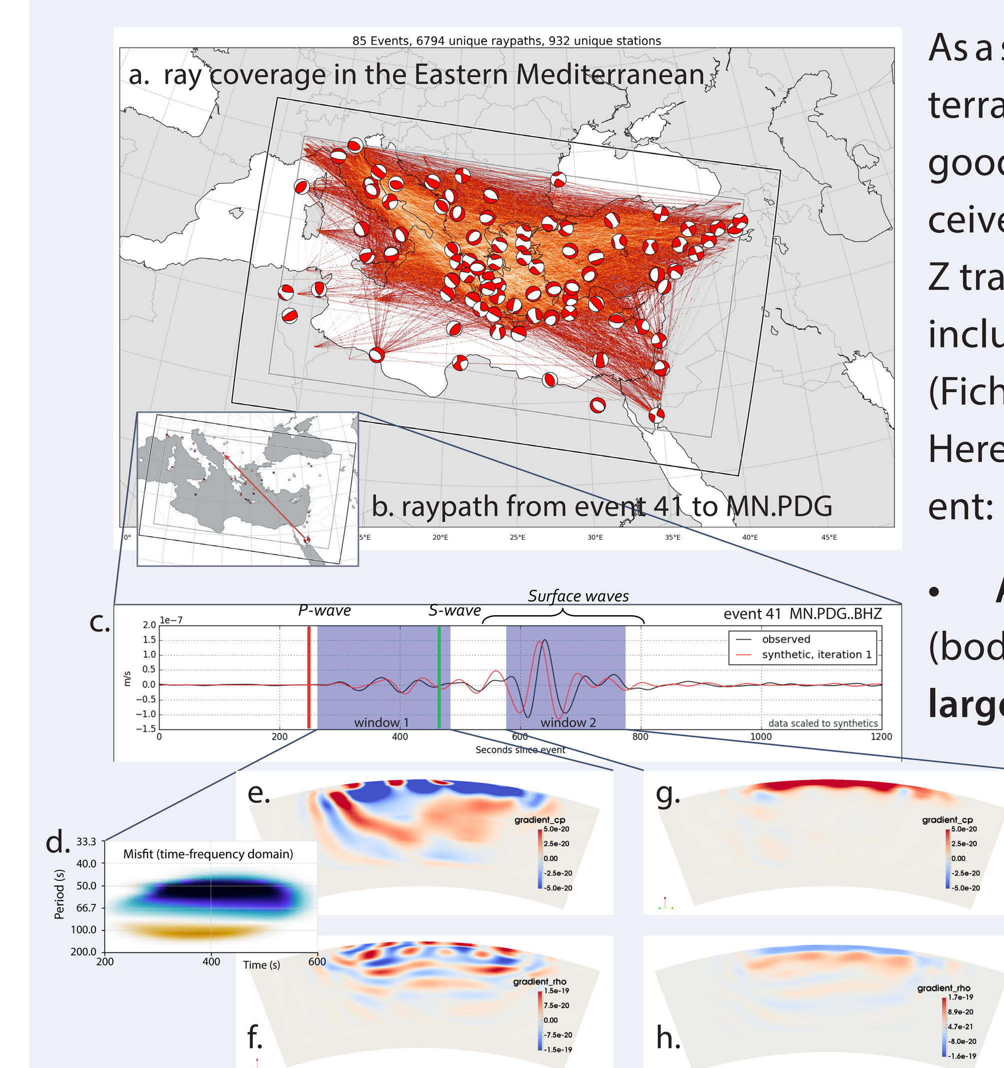


Figure 6. (a) Data coverage. (b) Raypath for trace in (c). (c) seismogram for event 41, station MN.PDG (Z component). (d) instantaneous phase shift within window 1 as a function of time and frequency. (e-h) sensitivity kernels for P velocity and density.

As a study area, we chose the Eastern Mediterranean, a tectonically active region with good data coverage. For one source-receiver path, we show two windows on the Z trace, filtered between 50-150 s. We also include the time-frequency phase misfit (Fichtner et al, GJI 2008) sensitivity kernels. Here, a number of things become apparent:

- **Amplitudes are lower** in window 1 (body waves), but **sensitivity to density is larger** than in window 2 (surface waves).
- Sensitivity to density has a significantly **different pattern** from sensitivity to the other parameters.

For this reason, we will construct optimal observables for density using different windows per trace, and different frequency bands.

7. Conclusions

- **It is possible to invert for density** on a global scale using seismic waveform inversion.
- Ignoring density or scaling it to velocity results in artefacts and loss of valuable information.
- Density can still be recovered at **noise levels of ~5%** - similar to high-quality data.
- **Optimal observables** can serve as a method to further isolate the density effect.
- The use of this method depends critically on **subjective choice** of observables, and choice of optimality criterion.