Characteristics of colliding density currents: a numerical and theoretical study

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Background Motivation

Density currents are primarily horizontal flows generated by horizontal density gradient. Thunderstorm outflows and sea breezes, amongst the others, have been shown to resemble density currents features. The interaction and collision of these flows can trigger severe convection and lead to severe weather.

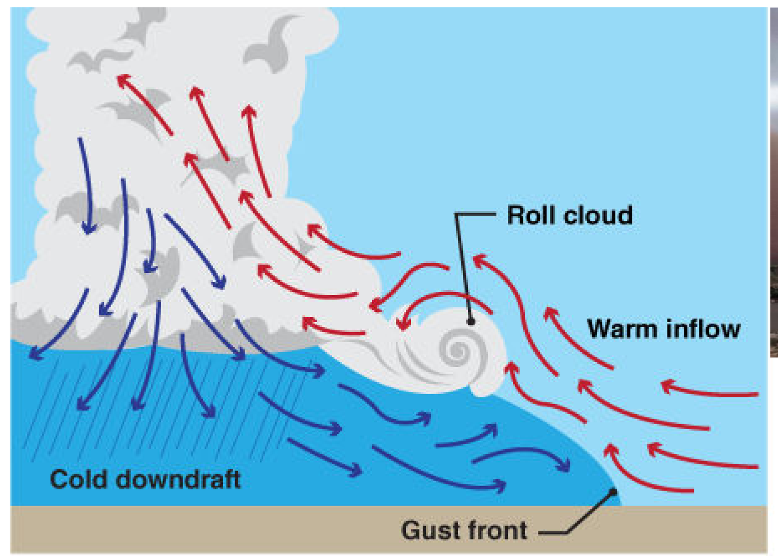

Figure 1. Schematic diagram of a thunderstorm outflow (left) and haboob in Texas on 18 June 2009 (right), source: Wikipedia.

Why this study ?

- **Convective initiation** and **cold pool parametrization** has not received a lot of attention in the prior literature. Parametrization is needed since GCM are too coarse to capture cold pool processes.
- **Reproduce numerically** the recent **laboratory experiments** of colliding density currents by van der Wiel et al. (2017) in an idealised setup.
- **Better understanding the dynamics** of the collision, in particular two features: maximum height reached by the fluid after collision and the **interface angle** between the two fluids. The latter in particular is **relevant for the position** and **strength of collision-induced convective triggering**.
- **Predict this interface angle**, prior to the running of the numerical model, based on the density ratio of the two fluids.

Numerical model

Initial conditions and setup

2D rectangular tank with height H and length L, with the two fluids with depth D at the two sides with buoyancies b_1 and b_2.

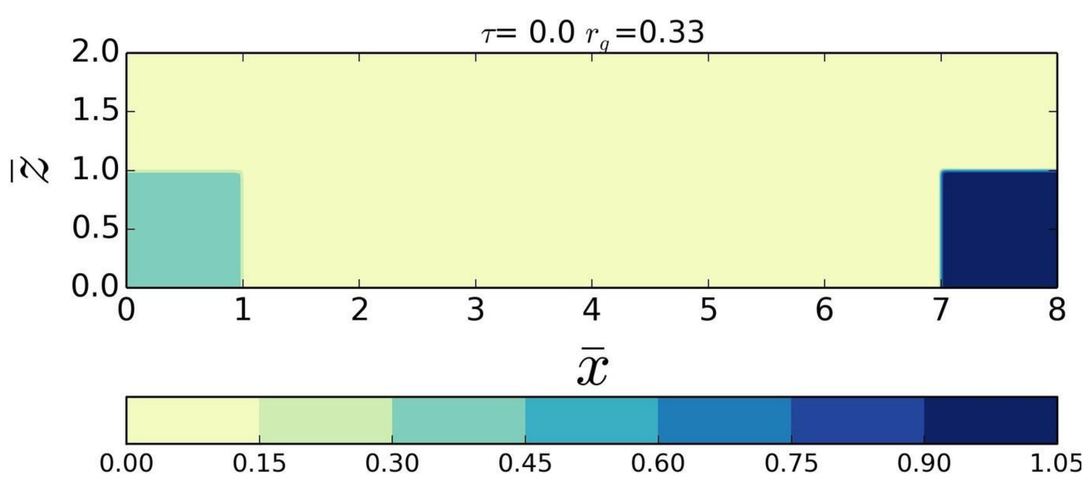

Figure 1. Numerical model domain at initial time. Color bar indicates buoyancy scale.

Numerical model and parameters

Using the numerical software package called Hydra (http://www-vortex.mcs.st-and.ac.uk) we solve **the Boussinesq approximation** of the **vorticity equations**. The numerical solution will depend on some **dimensionless parameters** (aspect ratio $\frac{H}{L}$, buoyancy ratio $r_g :=\frac{b_1}{b_2}$, depth ratio $\frac{D}{H}$). Therefore we have run **several experiments** varying these parameters.

$$\frac{D\bar{\eta}}{D\tau} = \frac{\partial \bar{b}}{\partial \bar{x}}$$

$$\frac{D\bar{b}}{D\tau} = 0$$

$$\nabla \cdot \bar{\mathbf{u}} = 0,$$

Experiment	D/H	l/L	H/L	r_g
1	1/2	1/8	1/8	0.99
2	1/2	1/8	1/8	0.75
3	1/2	1/8	1/8	0.49
4	1/2	1/8	1/8	0.33
5	1/2	1/8	1/8	0.25
6	1/2	1/8	1/8	0.20
7	1/2	1/8	1/8	0.15
8	1/2	1/8	2/8	0.99
9	1/2	1/8	2/8	0.75
10	1/2	1/8	2/8	0.49
11	1/2	1/8	2/8	0.33
12	1/2	1/8	2/8	0.25
13	1/2	1/8	2/8	0.20
14	1/2	1/8	2/8	0.15
15	1/2	1/8	2/8	0.05
16	1/4	1/8	4/8	0.99
17	1/4	1/8	4/8	0.75
18	1/4	1/8	4/8	0.49
19	1/4	1/8	4/8	0.20

$\bar{x} = x/l, \quad \bar{z} = z/l$

$\tau = t/\sqrt{l/|b_2|}$

$\bar{\eta} = \eta/\sqrt{|b_2|/l}$ where η is the only non-zero component of the vorticity field

$\bar{b} = b/|b_2|$

Numerical simulations

Animations of the simulations can be downloaded scanning the QR code provided.

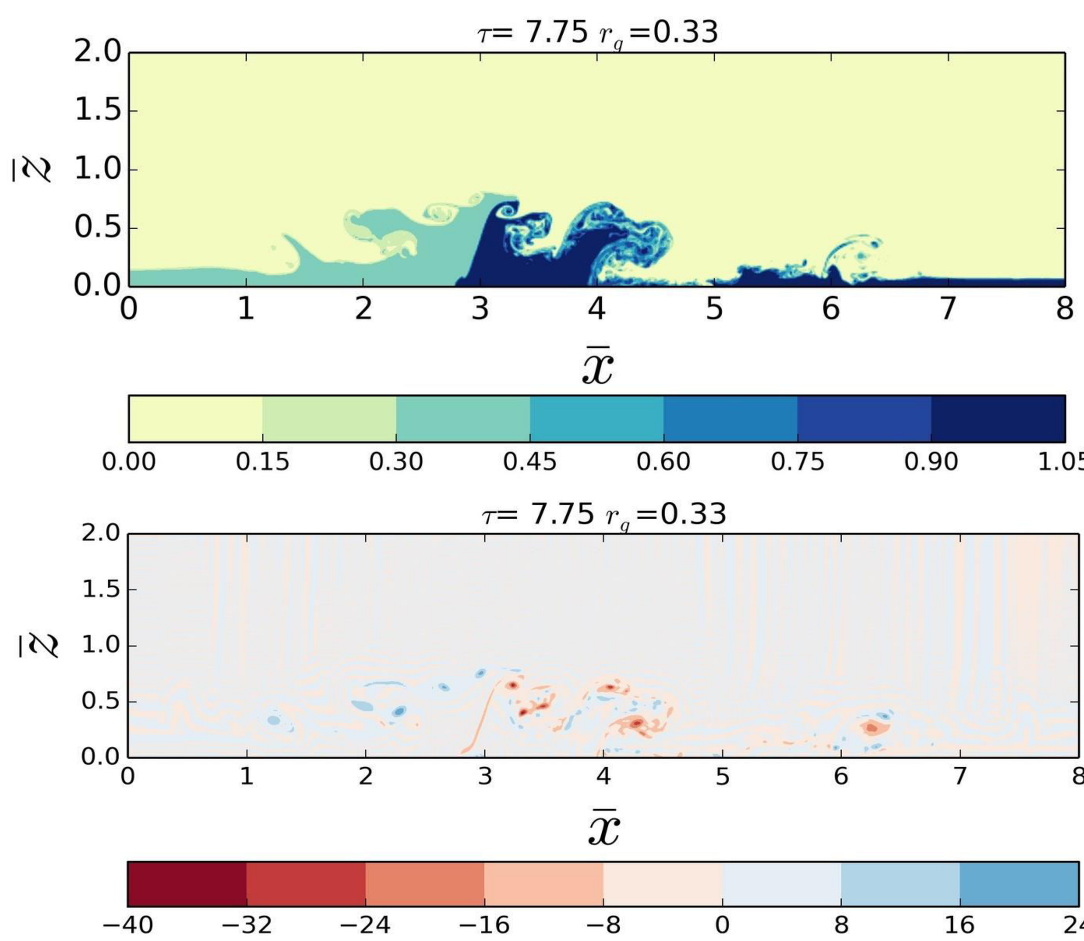

Figure 2. Snapshot of buoyancy (top) and vorticity field (bottom) model output at the moment of collision for experiment 11.

Theoretical model for interface angle

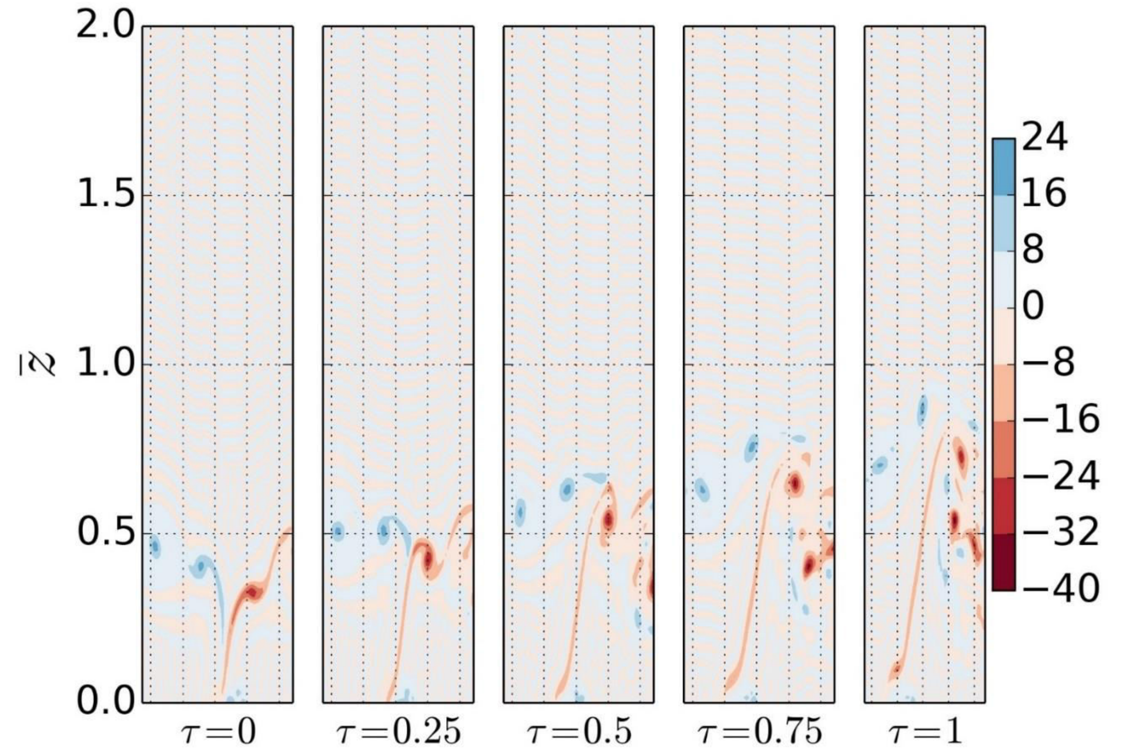

Figure 3. Time sequence of vorticity field during and after the collision for experiment 11.

The aim is to **understand** the role of **vorticity field** (plot shown on the left for r_g=0.33. The denser current (marked by red vorticity field) intrudes underneath the less dense current (in blue). This dynamic can be **described by the angle** that the interface between the two fluids form with the horizontal bottom line.

Formation of **vortices with opposite sign circulation** at the head of current can be seen. Therefore we have modelled the collision dynamics as a vortex pair system. This analysis gives us a **predictive formula** for the **angle with the bottom horizontal line**.

$$\gamma = \frac{\pi}{2} - \beta = \left(\frac{\pi}{2} - k\right) + k\, r_g^{1/2}$$

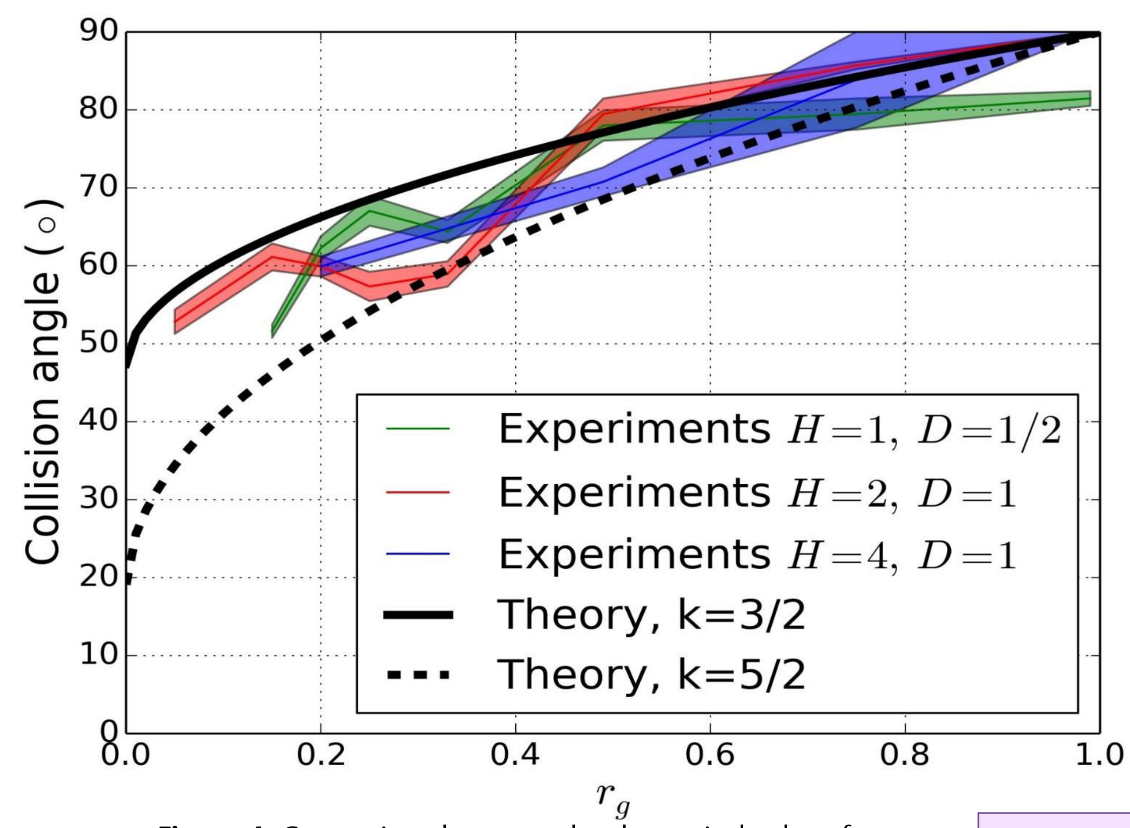

Figure 4. Comparison between the theoretical values for the angle given by the formula and numerical values.

The **theoretical value** for the angle **agrees well** with **numerical values** for all buoyancy ratios (see figure 4).

However this model is approximate, and is only expected to **apply for a short interval** during the collision. Nonetheless, it has **two implications**. The first, from is that, at a given dimensionless time after collision, the **angle is proportional** to the **net circulation** in the collision zone. The second, from is a **parabolic dependence** of the measured angle on the **buoyancy ratio**, with a vertical interface in the case r_g= 1.

Highlights, conclusions and future work

- The numerical simulations in idealized setup carried in this study represent a **first step towards convective cold pool parametrization**.
- The **vorticity Boussinesq model** is able to **capture** the essential **features of the collision dynamics.**
- The **predictive formula** for the interface angle **agrees well** with numerical simulations values.
- The present **study** could be **relevant** only to **daytime well mixed boundary layers**. Therefore **more simulations** are needed to explore more realistic situations (**stratification** and **wind shear**).

References

1. Cafaro C. and Rooney G.G., 2018. 'Characteristics of colliding density currents: a numerical and theoretical study', Quarterly Journal of the Royal Meteorological Society (in review)
2. van der Wiel K., Gille ST, Llewellyn Smith SG, Linden PF, Cenedese C. 2017. Characteristics of colliding sea breeze gravity current fronts: a laboratory study. Quarterly Journal of the Royal Meteorological Society 143(704): 1434–1441, doi:10.1002/qj.3015.

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