Dynamics of fragmented geomaterials

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Abstract

Fragmented geomaterials consist of blocks (fragments) that can move and rotate independent of each other. In some circumstances the relative movement of fragments is caused by external vibrations (e.g., waves) leading to fragment oscillation . Due to their limited movement, the neighbouring fragments can collide dissipating energy. When the fragments collide in the process of mutual rotation, the loss of energy falls to the neutral states, while the rest of the trajectory of oscillations can remain elastic.

In this study, forced oscillations of a pair of neighbouring fragments are analysed as a basic element of the process of fragment movement with energy dissipation on mutual collisions. The mathematical model of the interaction is represented as an undamped oscillator coupled with an additional condition: each time the system travels through the neutral points of the mass trajectory, the velocity is reduced by a coefficient of restitution (COR) smaller than one. As a result, the system transforms to piecewise linear with the non-linearity concentrated at the neutral points.

Numerical modelling shows that the behaviour of the model is influenced by three main parameters: the COR, the excitation frequency to natural frequency ratio, and the initial phase of the excitation. It was observed that the system can have periodic, asymmetric, and erratic non-periodic (chaotic) behaviour and energy dissipation does not always reduce with the decrease of COR. Also, it is found that, for odd super-harmonics, non-dissipative vibrations can occur either from the very beginning or after some stabilisation time.

Mathematical model

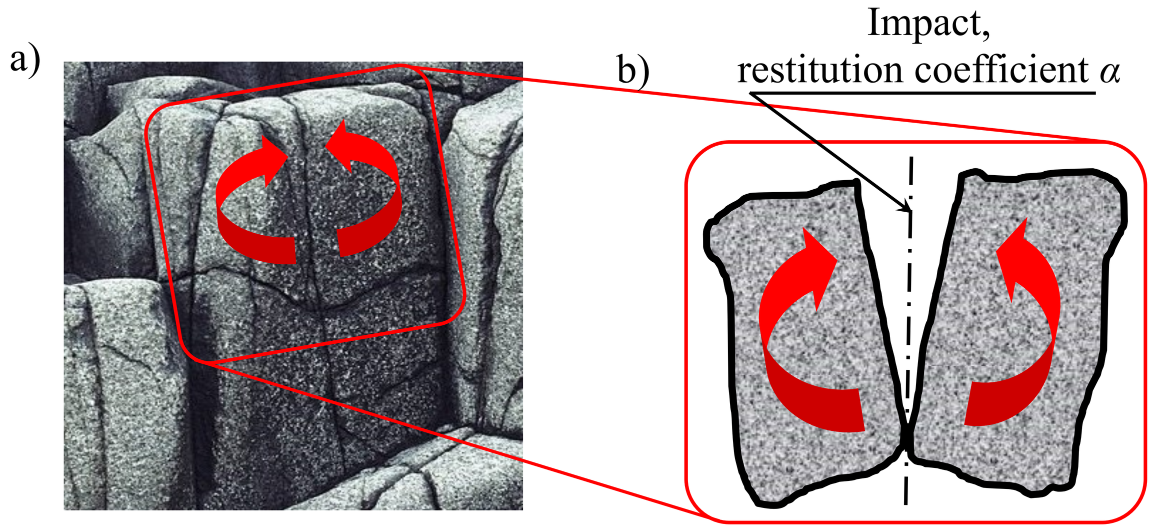


Fig. 1. a) Blocks of fragmented geomaterials, b) Rotational oscillations of the blocks (fragments)

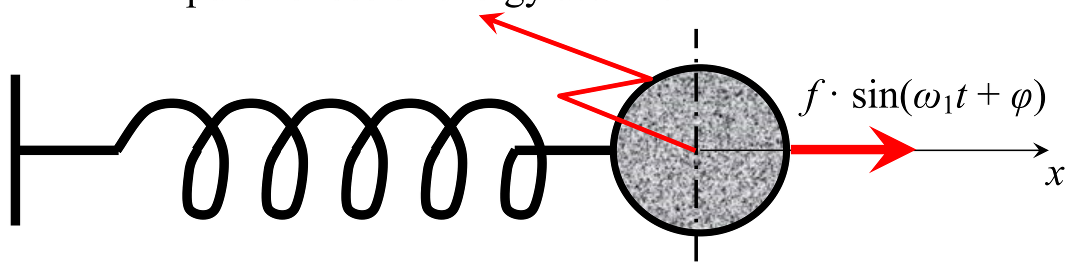


Fig. 2. Mathematical model of the oscillations in geomaterials

A system describing the oscillations of two blocks:

$$\begin{cases} \ddot{x} + \omega_0^2 x = f \cdot \sin(\omega_1 t + \varphi_0) \quad x(0) = 0, \quad v(0) = \dot{x}(0) = v_0 \\ v(\mathrm{T}_i + 0) = \alpha \cdot v(\mathrm{T}_i - 0) \quad \text{at} \quad x(\mathrm{T}_i) = 0 \end{cases} \quad (1)$$

Defining the following dimensionless groups

$$X = \frac{x}{l}, \quad \tau = \omega_0 t, \quad \Omega = \frac{\omega_1}{\omega_0}, \quad F = \frac{f}{l\omega_0^2}, \quad V_0 = \frac{v_0}{l\omega_0}$$

the system (1) can be rewritten in a dimensionless form:

$$\begin{cases} X'' + X = F \cdot \sin(\Omega\tau + \varphi_0) \quad X(0) = 0, \quad V(0) = X'(0) = V_0 \\ V(T_i + 0) = \alpha \cdot V(T_i - 0) \quad \text{at} \quad X(T_i) = 0 \end{cases} \quad (2)$$

Algorithm of finding the solution

The solution of the system (2) without energy loss, i.e. $\alpha = 1$:

$$X(\tau) = -\frac{F}{(1-\Omega^2)}\left(\Omega\cos(\varphi_0)\sin(\tau) + \sin(\varphi_0)\cos(\tau)\right) + \\ + \frac{F}{(1-\Omega^2)}\sin(\Omega\tau + \varphi_0) + X_0\cos(\tau) + V_0\sin(\tau) \quad (3)$$

It can be seen that there are six independent parameters describing the behaviour of the system: F, Ω, φ_0, τ, X_0, and V_0; moreover, the COR α changes the first derivative of the function after "impacts" leading to seven parameters overall. However, X_0 and V_0 characterise only the initial energy of the system and, similarly to force viscous damping vibrations, can be eliminated; the magnitude of the periodic force, F, plays a role of scalar multiplier and does not affect the pattern of vibrations. Thus, Eq. (3) can be rewritten as following:

$$X(\tau) = \frac{\sin(\Omega\tau + \varphi_0) - \Omega\cos(\varphi_0)\sin(\tau) - \sin(\varphi_0)\cos(\tau)}{1-\Omega^2} \quad (4)$$

The solution for other values of α is identical to Eq. (3) yet only for the time interval prior to the first intersection with the neutral axes T_1. After finding numerically T_1, we can solve the same Eq. (3) yet with different parameters φ_0, τ, and V_0 found from the solution of Eq. (3) at T_1.

The last procedure can be performed for all next parts of the solutions (Fig. 3) via the recurrent formula:

$$\begin{cases} X_{(i)}(\tau_{(i)}) = -\frac{\left(\Omega\cos(\varphi_{0(i)})\sin(\tau_{(i)}) + \sin(\varphi_{0(i)})\cos(\tau_{(i)})\right)}{(1-\Omega^2)} + \\ \quad + \frac{\sin(\Omega\tau_{(i)} + \varphi_{0(i)})}{(1-\Omega^2)} + V_{0(i)}\sin(\tau)_{(i)} \\ V_{0(1)} = 0, \quad \varphi_{0(1)} = \varphi_0, \quad \tau_{(i+1)} = \tau_{(i)} + T_i, \\ V_{0(i+1)} = \alpha V_{0(i)}(T_i), \quad \varphi_{0(i+1)} = \varphi_0 + \Omega T_i \end{cases} \quad (5)$$

The solution of the system (2) in a graphical form for different values of the major parameters are presented in Fig. 4-8.

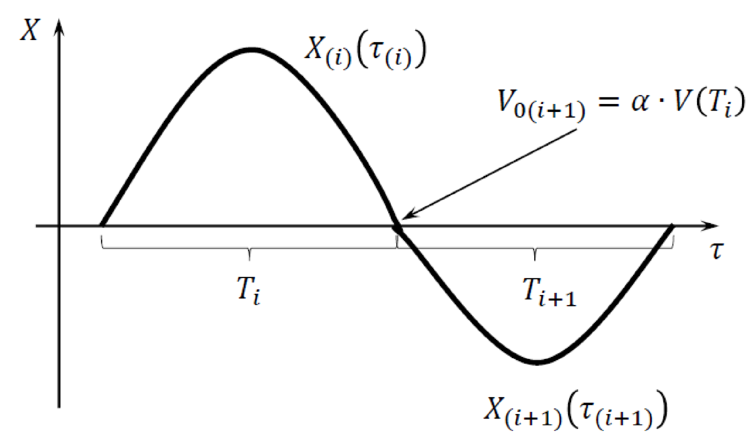
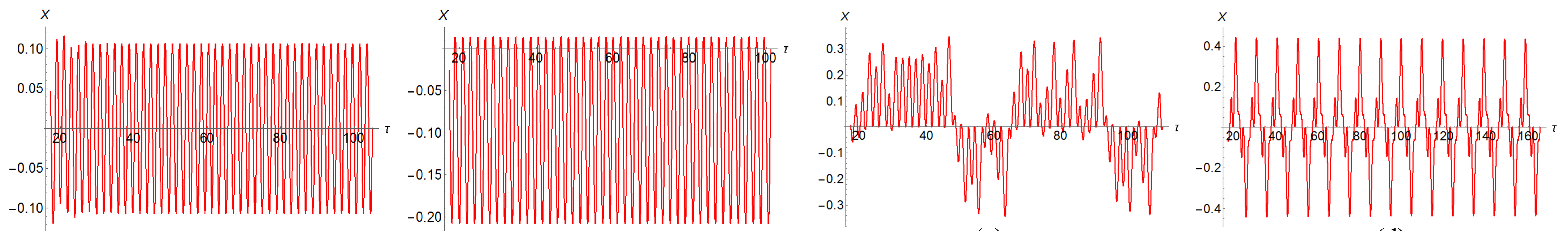


Fig. 3. Representing solution of Eq. (2) as a piecewise function

Using Eq. (5) one can prove that if Ω can be represented by $(2n+1)$ where n is a natural number, and $\varphi_0 = 0$, then the vibrations are periodic and without energy dissipation. (Fig. 6-7).

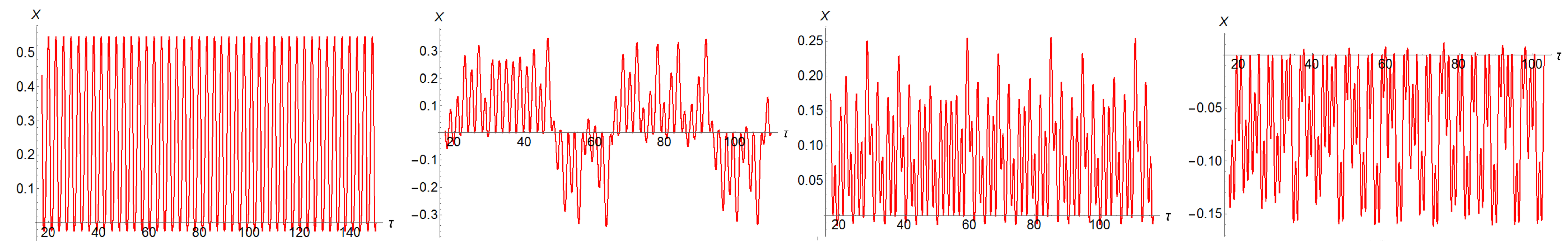
Also, for $\Omega = 3$ and $\alpha = 0$, there are a number of initial phases determined by the formula $\varphi_0 = 2^{-N}\pi Z$ (N and Z are natural and integer numbers, respectively) leading to undamped vibrations after some stabilisation time. (Fig. 8) The number of impacts to reach vibrations without energy loss cannot be greater than N.

Simulations



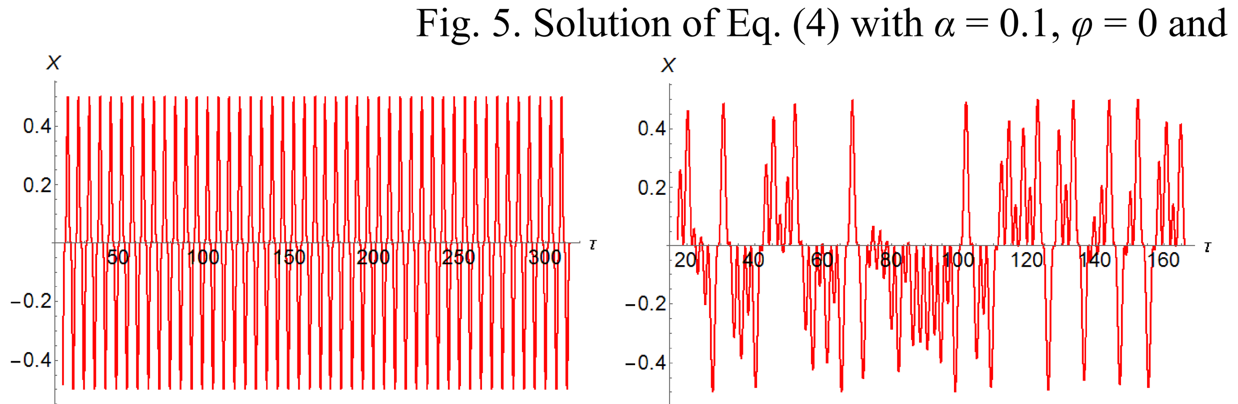
(a) (b) (c) (d)

Fig. 4. Solution of Eq. (4) with $\Omega = 3.2$, $\varphi = 0$ and different α: a) $\alpha = 0.8$, b) $\alpha = 0.3$, c) $\alpha = 0.1$, and d) $\alpha = 0.0$



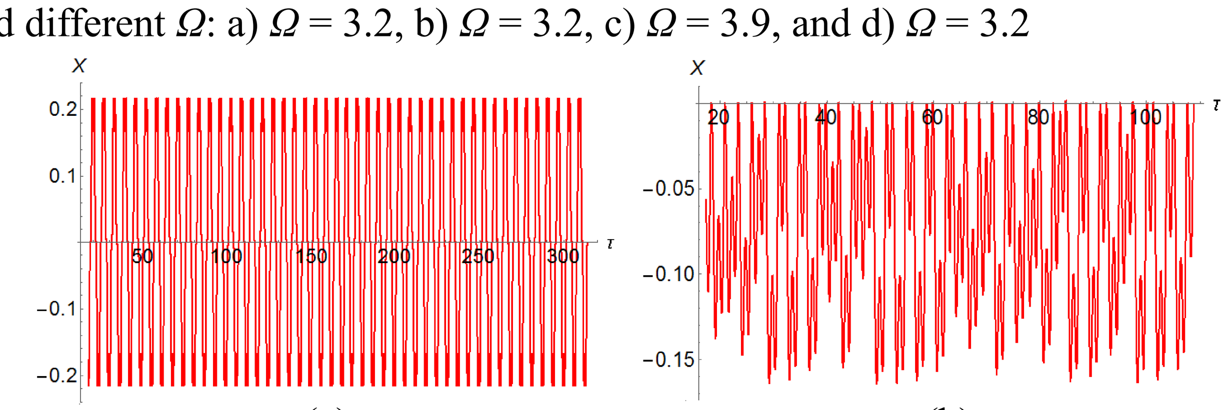
(a) (b) (c) (d)

Fig. 5. Solution of Eq. (4) with $\alpha = 0.1$, $\varphi = 0$ and different Ω: a) $\Omega = 3.2$, b) $\Omega = 3.2$, c) $\Omega = 3.9$, and d) $\Omega = 3.2$



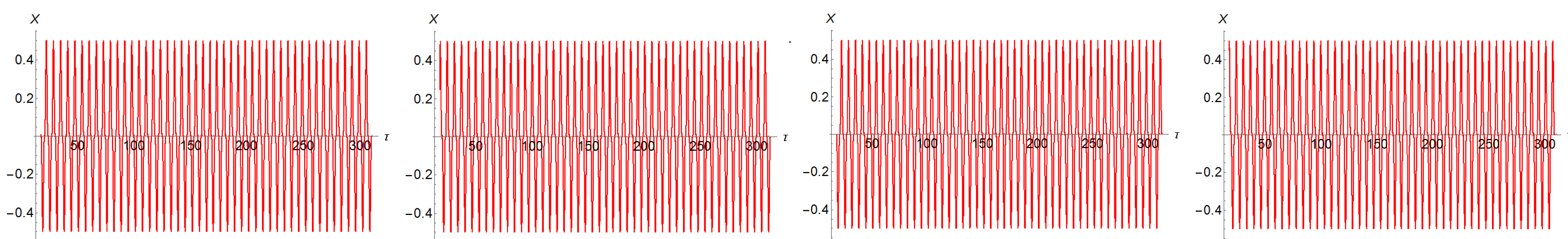
(a) (b)

Fig. 6. Solution of Eq. (4) with $\Omega = 5$, $\alpha = 0.1$ and a) $\varphi = 0$, b) $\varphi = \pi/90$



(a) (b)

Fig. 7. Solution of Eq. (4) with $\Omega = 3$, $\alpha = 0.2$ and a) $\varphi = \pi/4$, b) $\varphi = 23\pi/90$



(a) (b) (c) (d)

Fig. 8. Solution of Eq. (4) with $\alpha = 0.0$, $\Omega = 3$ and different φ: a) $\varphi = \pi/8$, b) $\varphi = \pi/4$, c) $\varphi = \pi/2$, and d) $\varphi = 5\pi/8$

Conclusions

- The basic element in the dynamics of fragmented geomaterials with mutual rotations is a linear oscillator with energy loss at neutral points (Fig. 2)
- Among six independent parameters describing the behaviour of an oscillator with impact damping, there are four three, i.e. φ_0, Ω, τ, and α, which have an influence on the character of steady-state oscillations
- The solution can be represented as a piecewise function consisting of linear solutions between time intervals T_i
- Asymmetric vibrations in the system are a transitional configuration to chaotic behaviour (Fig. 4-5)
- Odd super-harmonics, i.e. $\Omega = 3, 5, 7$, with zero initial phase demonstrate non-dissipative vibrations regardless of α (Fig. 6-7)
- For $\Omega = 3$, and zero coefficient of restitution α, if φ_0 can be represented by $2^{(-N)}\pi Z$ where N and Z are natural and integer numbers, respectively, the vibrations become non-dissipative after some stabilisation time which depends on the two numbers (Fig. 8)

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