Field Crop Irrigation–Multi-Objective Optimization and Sensitivity to Weather Forecast Accuracy

Theodor Bughici [1] Eran Tas [2] Erick Fredj [3] Naftali Lazarovitch [1]

[1]Ben-Gurion University of the Negev, Israel. [2]The Hebrew University of Jerusalem, Israel. [3]Lev Academic Center, School of Engineering and Computer Science, Israel.

Contact Information: Email: theodorb@post.bgu.ac.il Phone: +972 54 474 6536

Abstract

Accurate irrigation and fertigation of field crops are crucial for maximizing crop yield while avoiding overuse of water and fertilizer. Weather forecasts can predict potential evapotranspiration (ET_0) but are still far from perfect. We used two case study of sprinkler irrigated spring potatoes in Coastal Israel and summer peanuts in northern Israel test cases in order to define a minimal accuracy level of ET_0 predictions for irrigation planning. The working stages of simulation-optimization-sensitivity analysis are described in the workflow. By modeling crop irrigation based on varying forecasted ET_0 relative bias ranges as well as crop and soil parameters we were able to rank the parameters by contribution to crop-model output variance. Our main findings are:

- ET_0 prediction accuracy dominates the crop model parameters when ET_0 relative bias (δ_{ET_0}) range $< 5\%$ (case 1) and $< 2.5\%$ (case 2).
- Case1 - The soil n parameter dominates model output when δ_{ET_0} range $< 5\%$ for all objective functions but transpiration (RMSE$_{T_a}$).
- Case 2 - K_s dominates most objective functions and δ_{ET_0} ranges.
- For case 1 max. root depth is dominating transpiration output (RMSE$_{T_a}$) when δ_{ET_0} range $<$ 5%, while case 2 is dominated by the soil hydraulic parameters.

This procedure of optimization and sensitivity analysis can be extended to a wide range of case studies and help define what is an adequate weather forecast accuracy suitable to base crop irrigation upon.

Workflow

Simulation
Root water and solute uptake (HYDRUS-1D) of sprinkler irrigated potatoes in Coastal Israel.

→ 15,000 model runs →

Optimization
Finding optimal irrigation management, using a multiobjective genetic algorithm (NSGA*ii*).

→ 11,700 model runs →

Sensitivity analysis
Fourier amplitude sensitivity analysis (FAST) of model parameters with decreasing δ_{ET_0} ranges.

Simulation

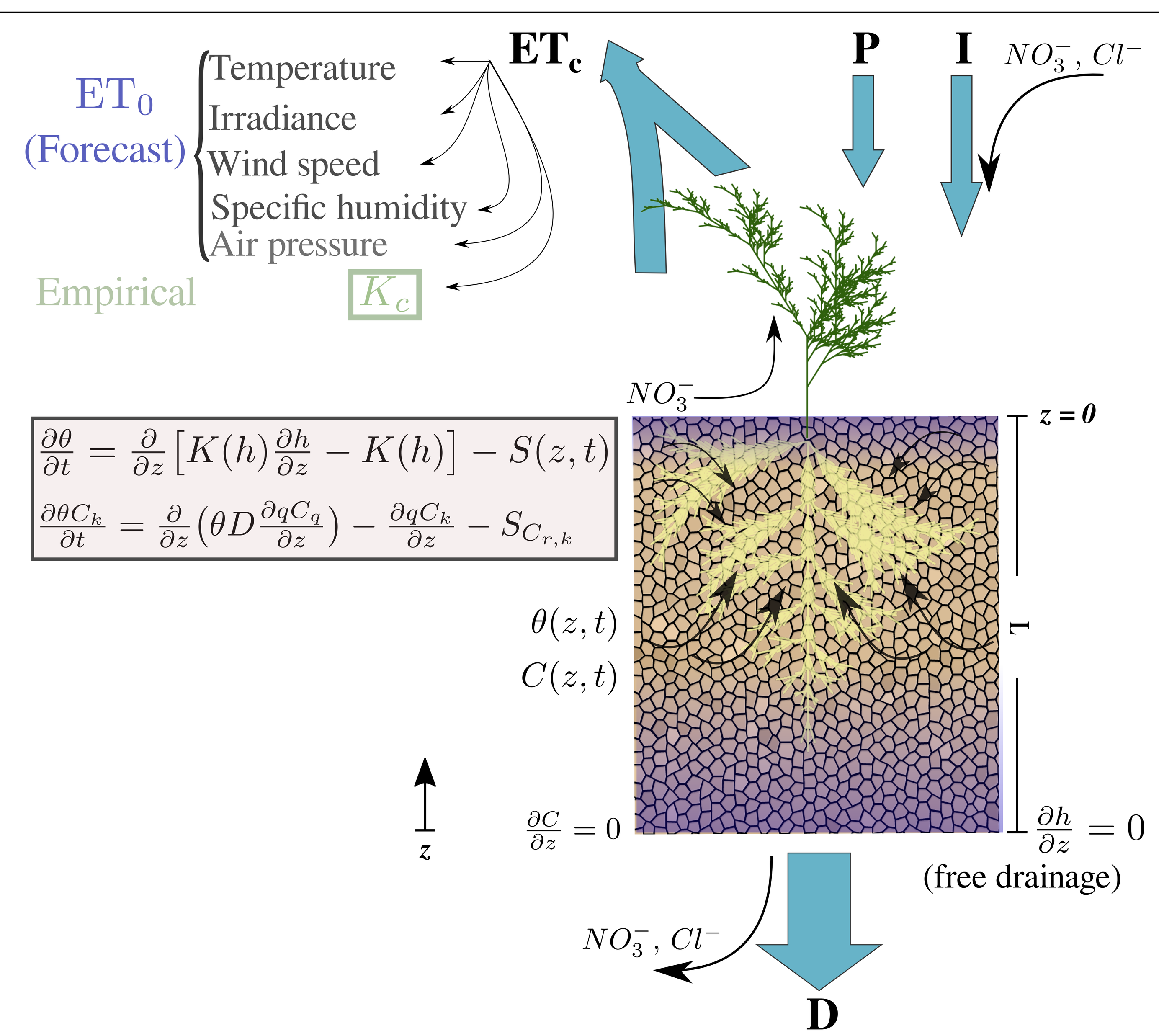

Figure 1. Crop model scheme. 1D water and solute movement and root uptake.

References

[1] Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: Nsga-ii. *IEEE Trans. Evol. Comput.*, 6(2):182–197.

[2] Saltelli, A., Tarantola, S., and Chan, K.-S. (1999). A quantitative model-independent method for global sensitivity analysis of model output. *Technometrics*, 41(1):39–56.

[3] Šimůnek, J., Šejna, M., Saito, H., Sakai, M., and van Genuchten, M. T. (2008). The hydrus-1d software package for simulating the movement of water, heat, and multiple solutes in variably saturated media, version 4.0: Hydrus software series 3. *Department of Environmental Sciences, University of California Riverside, Riverside, California, USA*, 315.

JCT LEV ACADEMIC CENTER Ben-Gurion University of the Negev THE HEBREW UNIVERSITY OF JERUSALEM

Optimization

The irrigation management multi-objective optimization problem is formulated as:

$$\begin{aligned} &optimize\ F(x) = (f_1(x), f_2(x), f_3(x)) \\ &subject\ to\ x \in \Omega \end{aligned} \quad (1)$$

Where Ω is the decision space. Pareto-optimal set is a solution if it is not dominated by any other solution in the decision variable space. For a given multi-objective problem, the Pareto-optimal set, $P^\star$, is defined as:

$$P^\star = \{x \in \Omega \,|\, \neg\exists x' \in \Omega \;\; F(x') \preceq F(x)\} \quad (2)$$

Case 1 - Spring potatoes irrigation in loamy sand Optimal irrigation & sensitivity analysis

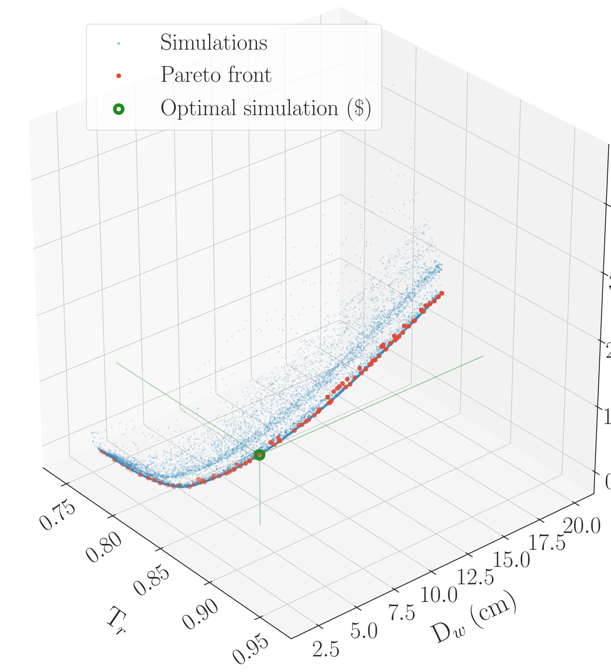

Figure 2. Objective space of the Max–Min–Min optimization. The objective functions are relative transpiration (T_r), water drainage (D_w) and solute drainage (D_s). The economically optimal ($) objective vector is marked in green. Optimal decision variables were: Irrigation rate = 14 $mm\,h^{-1}$, $\delta_{irrigation}$ = -1.9%, irrigation interval = 48 h with first irrigation after 3 h.

Table 1. Crop model input parameters tested for sensitivity and their ranges (case 1).

par.	description	range	par.	description	range
K_s	hydraulic conductivity (cm h^{-1})	5.7, 14.3	$Z_{r,\,max}$	max. root depth (cm)	40, 80
α	retention curve par. (cm^{-1})	0.04, 0.2	C_{50}	osmotic stress par. (dS m^{-1})	10.7, 14
n	retention curve par. (-)	1.94, 2.65	δ_{rain}	rain relative bias (%)	-100, 100
$h_{3,\,L}$	water stress par., low ET_0 (cm)	-175, -500	δ_{ET_0}	predicted ET_0 bias (%)	changing
$h_{3,\,H}$	water stress par., high ET_0 (cm)	-10, -175			

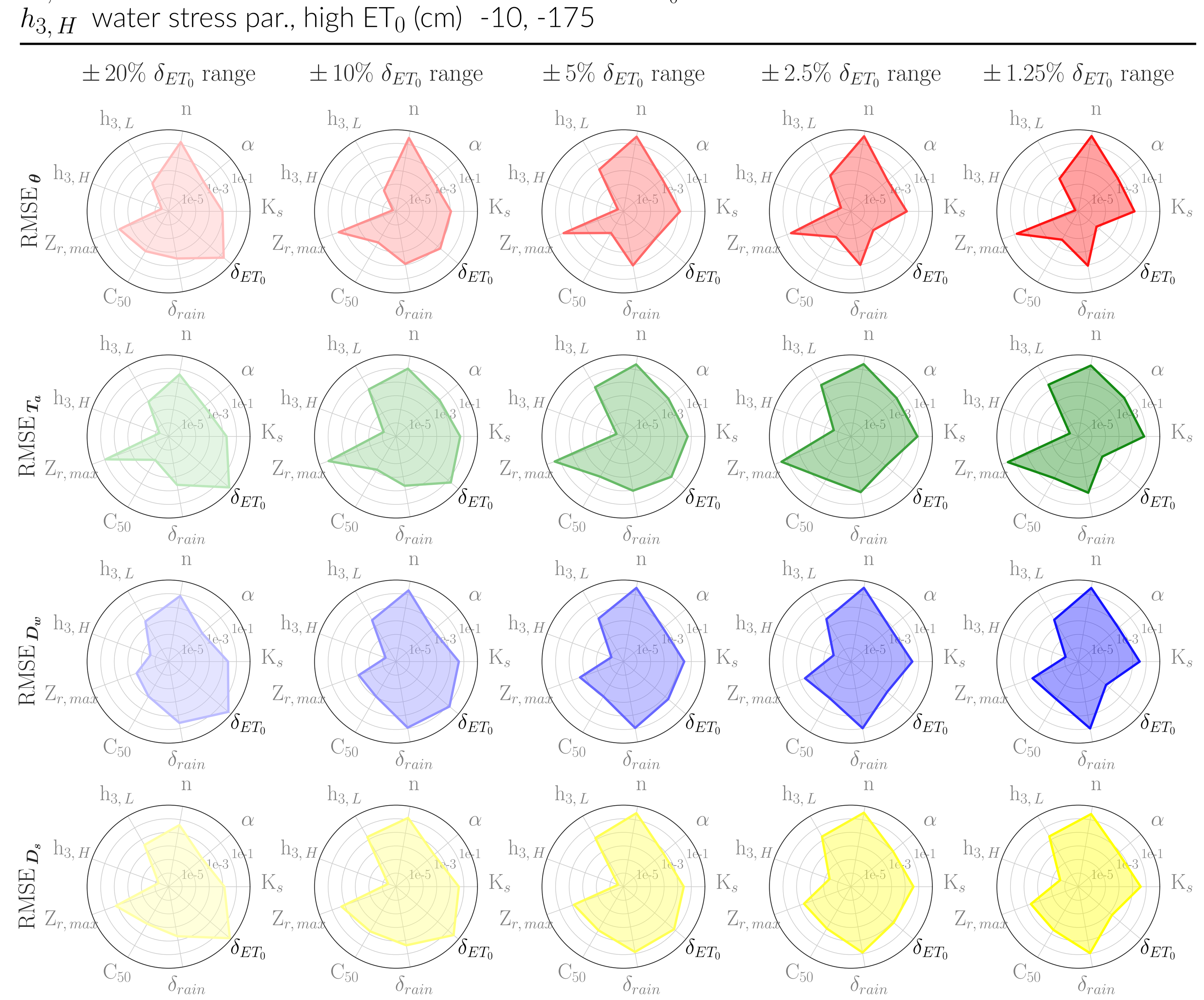

Figure 3. Total $\log_{10}$-sensitivity indices (S_{T_i}) of principal crop model parameters. The objective functions tested were the root mean squared error (RMSE) of the soil water content (θ) at seven depths (-10 to -70 cm), the actual transpiration (T_a), the water drainage (D_w) and solute drainage (D_s).

Sensitivity analysis

FAST is a variance-based global sensitivity test where $V = \sum_{i=1}^{k} V_i + \sum_{i=1}^{k}\sum_{j>i}^{k} V_{ij} + ... + V_{1,2,...,k}$ with V being the total variance of the model output, V_i is the first-order variance for each factor x_i and V_{ij} is interactions among k factors. The first-order sensitivity index corresponding to the parameter x_i is: $S_i = V[E(Y|x_i)]/V(Y)$ and the total-order sensitivity index of a single parameter (index i) with the interaction of more parameters that involve index i and at least one index $j \neq i$ from 1 to k is:

$$S_{T_i} = \sum S_i + \sum_{j\neq i} S_{ij} + ... + S_{1...k} \quad (3)$$

Case 2 - Summer peanuts irrigation in loamy soil Optimal irrigation & sensitivity analysis

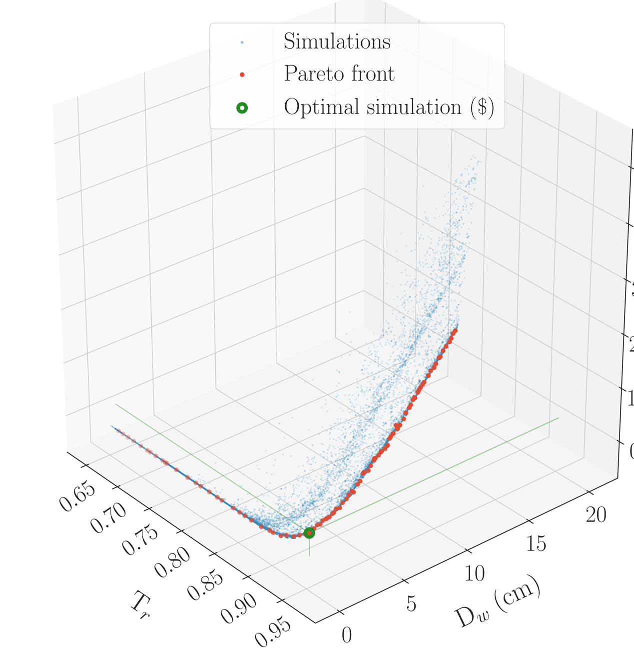

Figure 4. Objective space of the Max–Min–Min optimization. The objective functions are relative transpiration (T_r), water drainage (D_w) and solute drainage (D_s). The economically optimal ($) objective vector is marked in green. Optimal decision variables were: Irrigation rate = 14 $mm\,h^{-1}$, $\delta_{irrigation}$ = +19%, irrigation interval = 120 h with first irrigation after 24 h.

Table 2. Crop model input parameters tested for sensitivity and their ranges (case 2).

par.	description	range	par.	description	range
K_s	hydraulic conductivity (cm h^{-1})	0.37, 2.57	$Z_{r,\,max}$	max. root depth (cm)	50, 100
α	retention curve par. (cm^{-1})	0.01, 0.05	C_{50}	osmotic stress par. (dS m^{-1})	6.8, 12.7
n	retention curve par. (-)	1.23, 1.88	δ_{rain}	rain relative bias (%)	-100, 100
$h_{3,\,L}$	water stress par., low ET_0 (cm)	-500, -2000	δ_{ET_0}	predicted ET_0 bias (%)	changing
$h_{3,\,H}$	water stress par., high ET_0 (cm)	-10, -500			

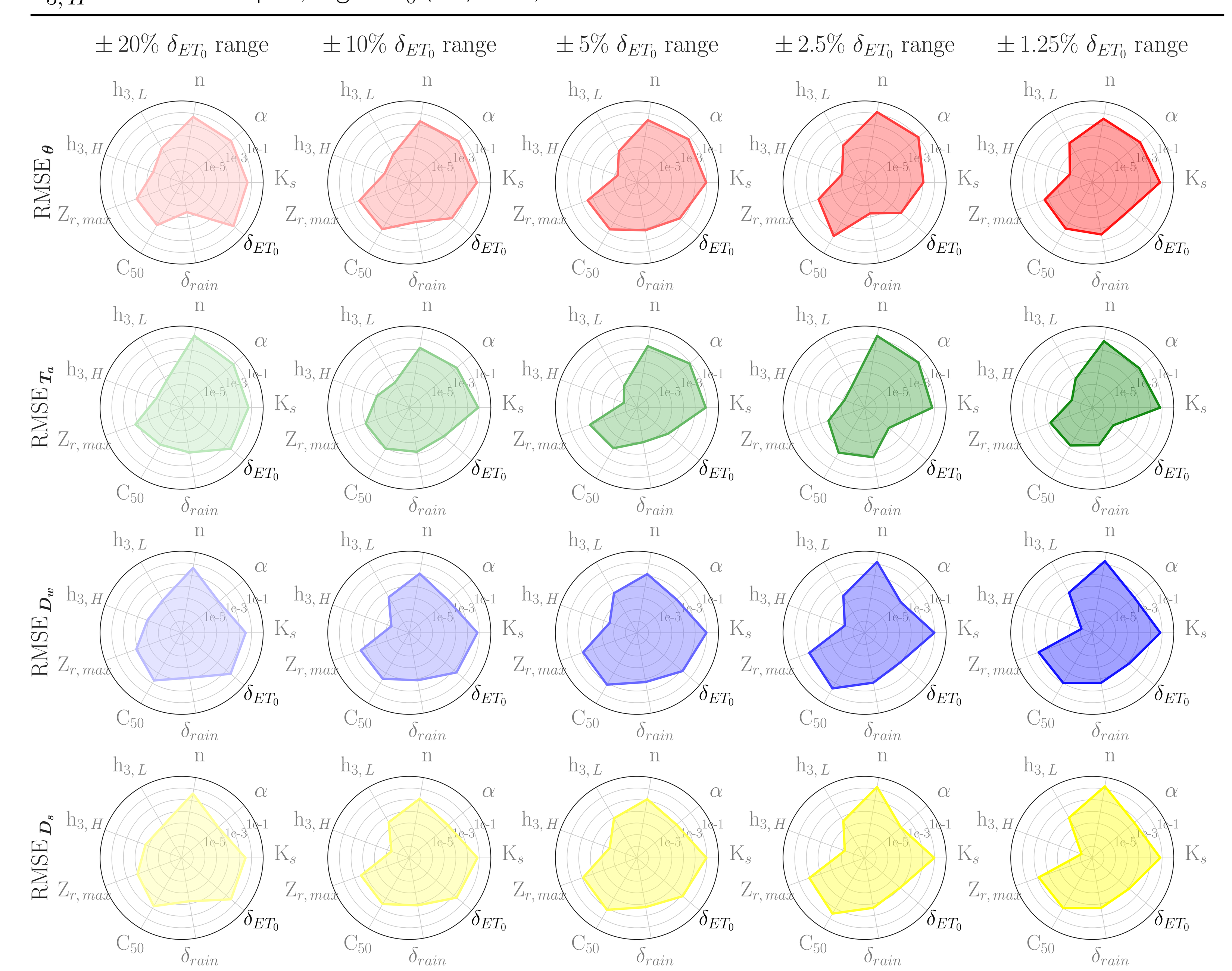

Figure 5. Total $\log_{10}$-sensitivity indices (S_{T_i}) of principal crop model parameters. The objective functions tested were the root mean squared error (RMSE) of the soil water content (θ) at seven depths (-10 to -70 cm), the actual transpiration (T_a), the water drainage (D_w) and solute drainage (D_s).