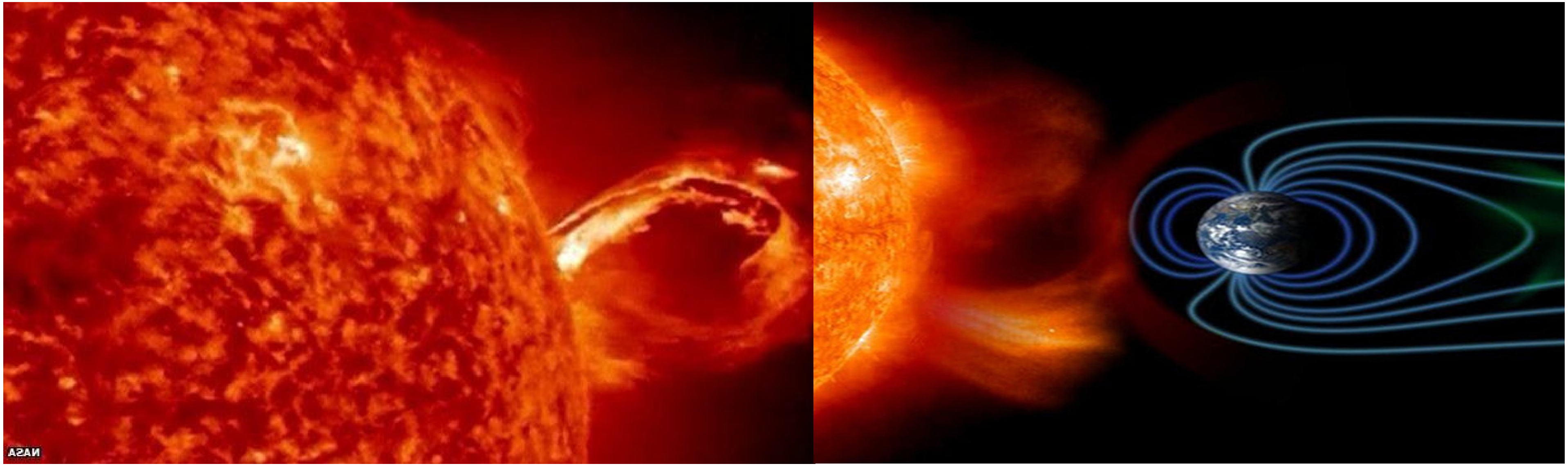
Reconstructing the dynamics of the outer electron radiation belt by means of the standard and ensemble Kalman filter with the VERB-3D code.



Credit: NASA

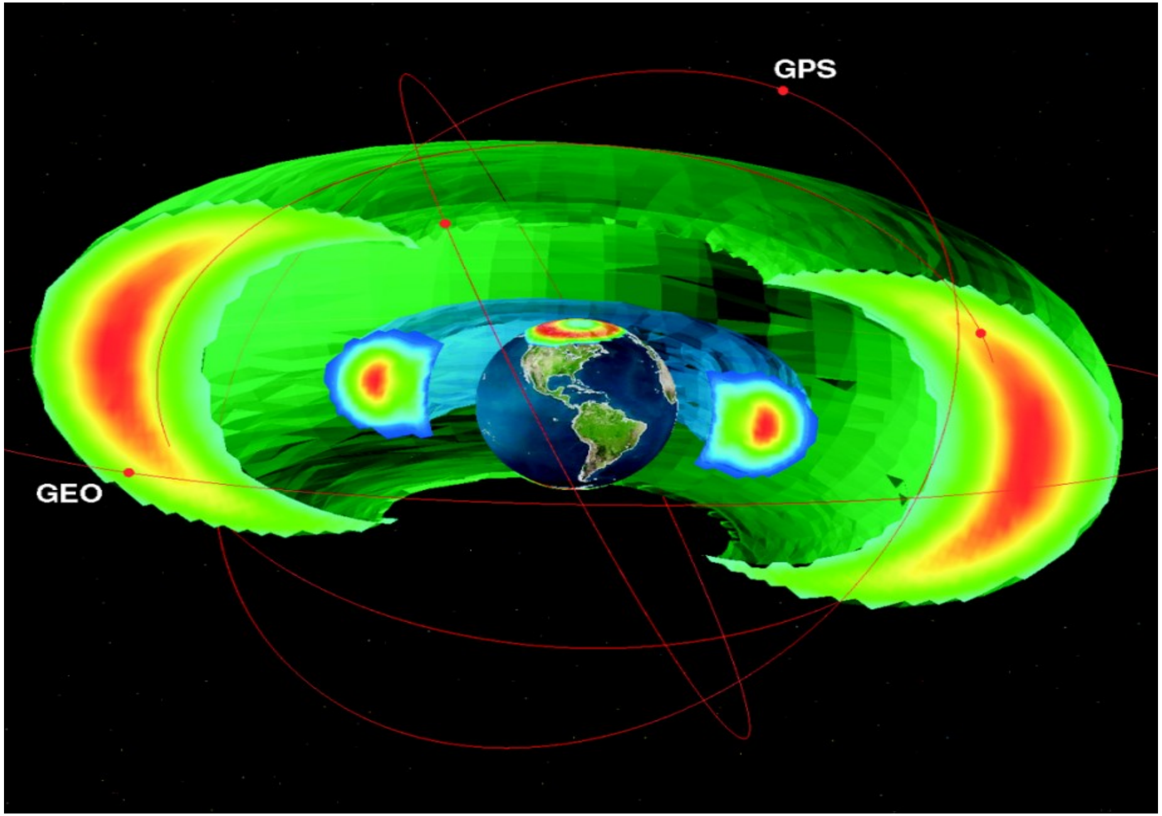
Authors:

Angelica M. Castillo, Jana de Wiljes, Yuri Shprits, Nikita Aseev

EGU - General Assembly, 23-27 Mai, 2022

Introduction: The Radiation belts or Van Allen belts

- Particular focus on electron populations in the radiation belt region.



Credit: SWPC-NOAA

Radiation belts:

two donut shaped regions of high radiation encompassing the Earth.

- energies >100 keV
- two-zone structure

Inner belt: mostly protons, fairly stable.

Outer belt: mostly electrons, can change on the time scales of minutes to an hour.

Introduction: Model and data sources

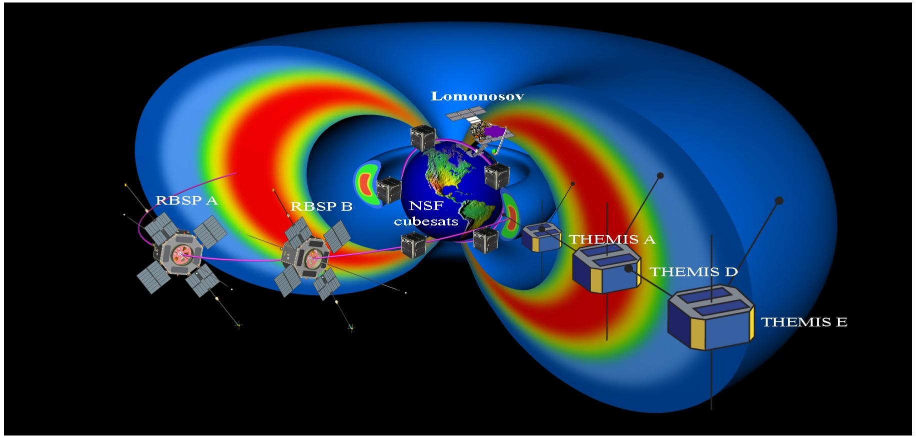
Model: VERB-3D code

Phase Space Density (PSD) = *f*

$$
\begin{aligned}
\frac{\partial f}{\partial t} = {} & L^{*2} \frac{\partial}{\partial L^*}\bigg|_{J_1,J_2} \frac{1}{L^{*2}} D_{L^*L^*} \frac{\partial f}{\partial L^*}\bigg|_{J_1,J_2} + \\
& + \frac{1}{p^2}\frac{\partial}{\partial p}\bigg|_{L,\alpha_0} p^2 \left(D_{pp}\frac{\partial f}{\partial p}\bigg|_{L,\alpha_0} + D_{p\alpha_0}\frac{\partial f}{\partial \alpha_0}\bigg|_{L,p} \right) + \\
& + \frac{1}{T(\alpha_0)\sin(2\alpha_0)}\frac{\partial}{\partial \alpha_0}\bigg|_{L,p} T(\alpha_0)\sin(2\alpha_0) \left(D_{\alpha_0 p}\frac{\partial f}{\partial p}\bigg|_{L,\alpha_0} + D_{\alpha_0\alpha_0}\frac{\partial f}{\partial \alpha_0}\bigg|_{L,p} \right) + \\
& + Sources - Losses
\end{aligned}
$$

L- radial distance, *p* - momentum, a pitch-angle (angle between the magnetic field and particle momentum)

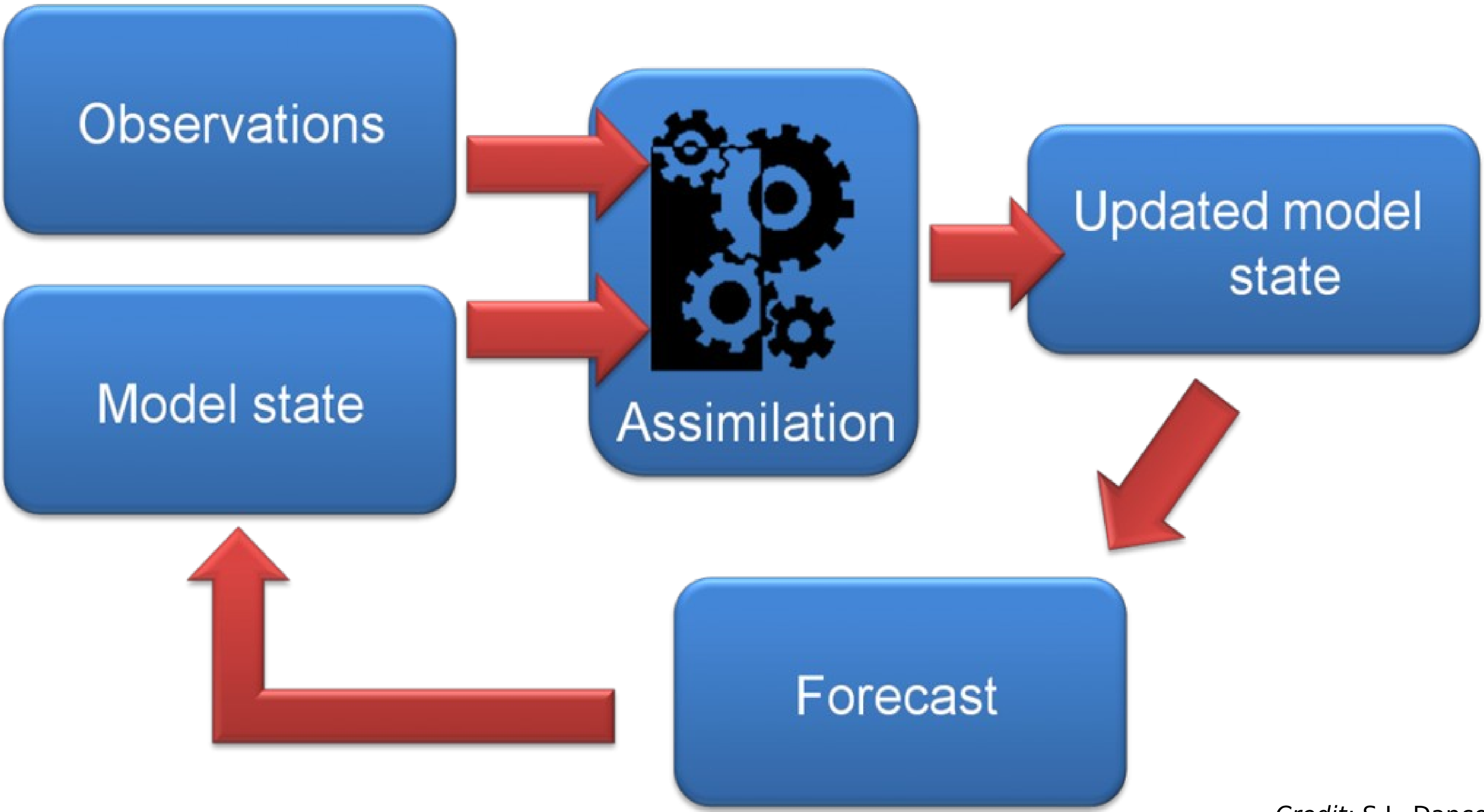
Data sources: Satellite measurements



Credit: NASA

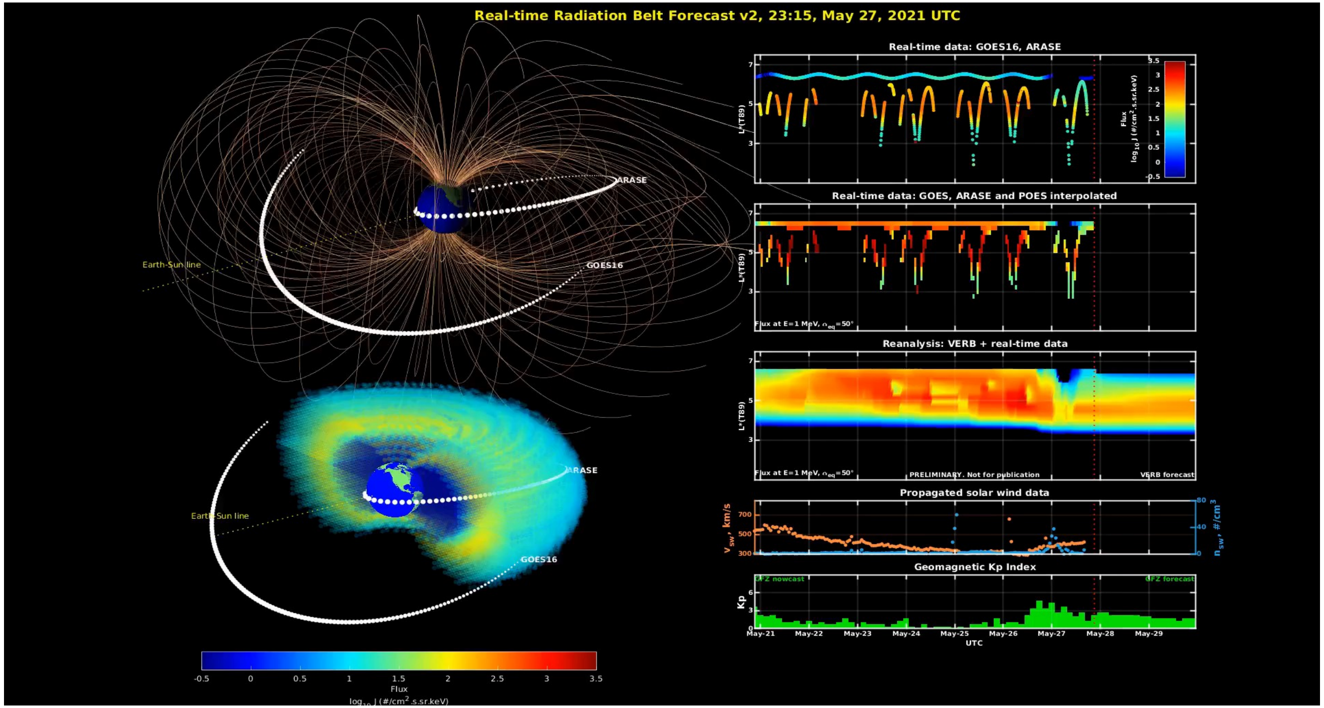
- Electron flux measurements from ongoing or past satellite missions, s.a. Van Allen Probes, GOES, ARASE, POES, etc.
- Need to utilize vast amount of data, including pitch-angle and energy distributions

Data Assimilation: Kalman Filtering



Credit: S.L. Dance. University of Reading, 2022.

Previous work: Implementation of the standard Kalman filter



- Implementation of the KF for the radiation belt system performed by e.g. Shprits et al. 2013, Kellermann et al. 2014
- Long-term reconstruction of the near Earth radiation environment (Cervantes et al., 2020)
- Initial DA studies validated our current knowledge of the system and initial parameter estimation showed the need to further improve our empirical models.

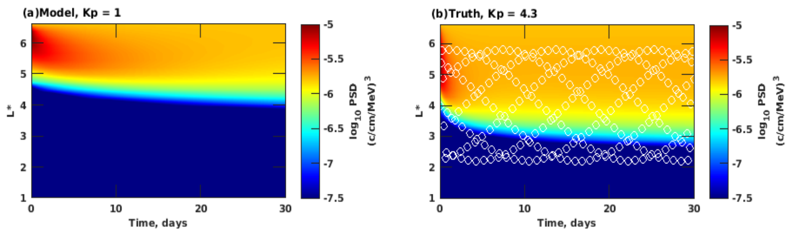
GFZ- Section 2.7: https://isdc.gfz-potsdam.de/data-assimilative-radiation-belt-forecast/

Set-up of the EnKF: standard KF vs. EnKF

Filter Equations	Standard KF	EnKF
Forecast state	$\psi_{k+1} = F\psi_k,$	$\psi_{i,k}^f = \mathbf{M}\psi_{i,k-1}^a + d\mathbf{q}_i^k,$
Error covariance matrix of the model	$\mathbf{Q}_{KF} = (\sigma \cdot \psi_f)^2$	$\boldsymbol{Q}_e = \overline{d\boldsymbol{q}_k d\boldsymbol{q}_k^T},$
Observations	$\boldsymbol{d} = \boldsymbol{H}\psi^t + \epsilon.$	$\boldsymbol{d}_j = \boldsymbol{d} + \epsilon_j,$
Error covariance matrix of the observations	$\mathbf{R}_{KF} = (\sigma \cdot \psi_o)^2$	$\boldsymbol{R}_e = \overline{\epsilon\epsilon^T},$
Error covariance matrix of the forecast	$\boldsymbol{P}_{k+1} = \boldsymbol{F}\boldsymbol{P}_k\boldsymbol{F}^T + \boldsymbol{Q},$	$\boldsymbol{P}^f \simeq \boldsymbol{P}_e^f = \overline{(\psi^f - \overline{\psi^f})(\psi^f - \overline{\psi^f})^T},$
Error covariance matrix of the analysis	$\boldsymbol{P}^a = \boldsymbol{P}^f - \boldsymbol{P}^f\boldsymbol{H}^T(\boldsymbol{H}\boldsymbol{P}^f\boldsymbol{H}^T + \boldsymbol{R})^{-1}\boldsymbol{H}\boldsymbol{P}^f.$	$\boldsymbol{P}^a \simeq \boldsymbol{P}_e^a = \overline{(\psi^a - \overline{\psi^a})(\psi^a - \overline{\psi^a})^T},$
Analysis state	$\psi^a = \psi^f + \boldsymbol{P}^f\boldsymbol{H}^T(\boldsymbol{H}\boldsymbol{P}^f\boldsymbol{H}^T + \boldsymbol{R})^{-1}(\boldsymbol{d} - \boldsymbol{H}\psi^f),$	
Gain matrix	$\boldsymbol{K} = \boldsymbol{P}^f\boldsymbol{H}^T(\boldsymbol{H}\boldsymbol{P}^f\boldsymbol{H}^T + \boldsymbol{R})^{-1}.$	

Synthetic experiment: Convergence of EnKF to KF

Test of setup and convergence of the EnKF in a controlled environment using synthetic data.

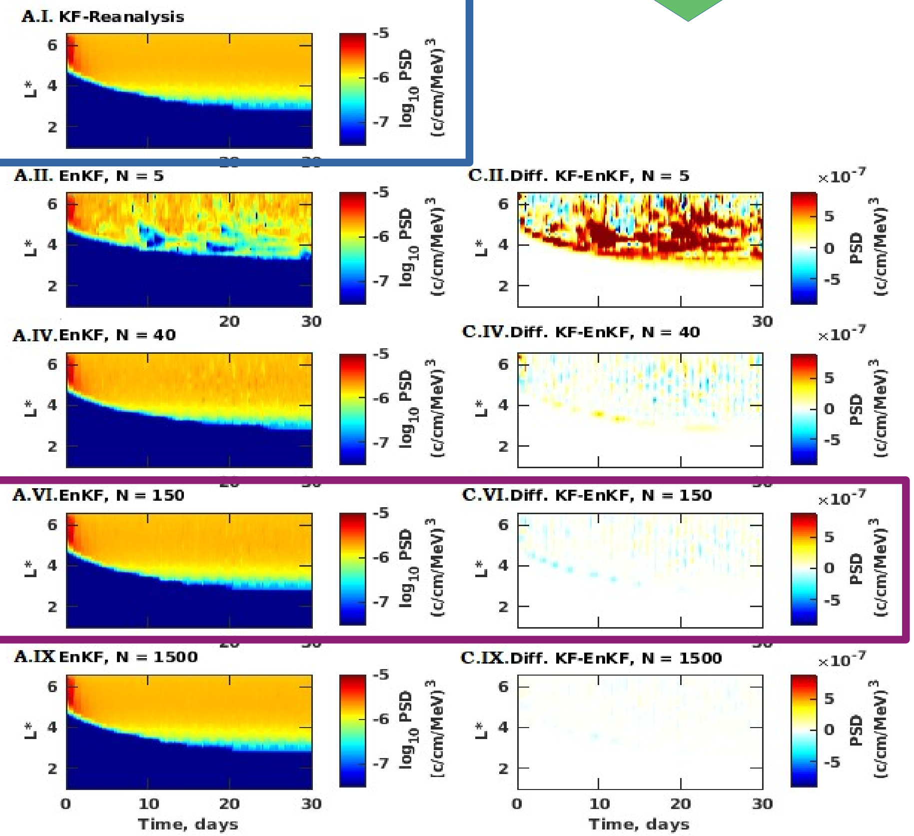


EnKF converges to KF solution with increasing number of ensemble members

150 Ensemble members enough to reproduce KF solution

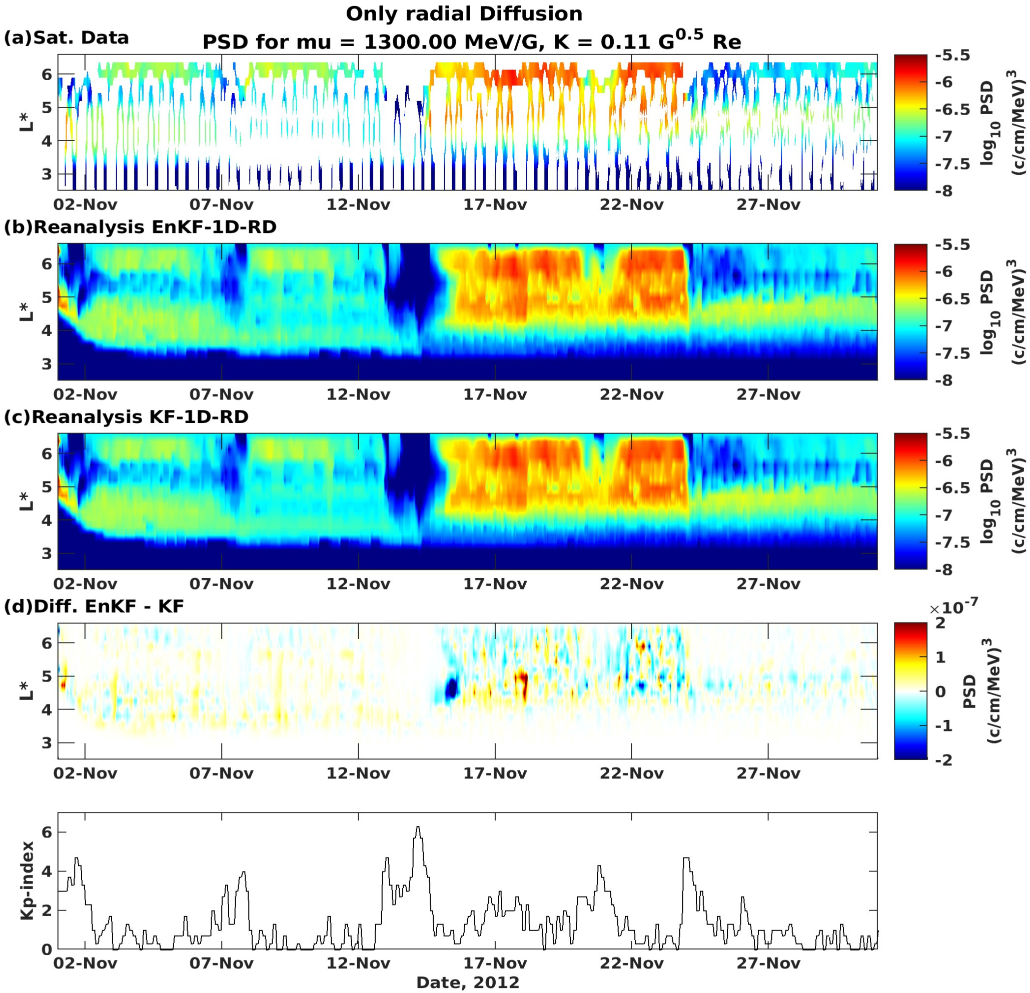
Assimilation results

Difference between KF and EnKF results



Castillo et al. 2021, Space Weather

DA with real-data: Standard KF vs. EnKF



Raw satellite data

EnKF with 150 Ensembles

Kalman Filter

PSD Difference: EnKF - KF

Castillo et al., 2021, Space Weather

Conclusions and Outlook

GFZ
Helmholtz-Zentrum
POTSDAM

Conclusions:

- We have successfully implemented the KF and EnKF for the radiation belt region
- 150 ensemble members are sufficient for convergence to the optimal state solution of the KF.
- Assimilation of real satellite data the KF and EnKF delivers similar results.

Future Work:

- Improve grid resolution, a higher dimensional setup may require localization or inflation
- Model bias, model error, parameter estimation
- Improvement of initial ensemble
- Implementation of other ensemble based filters, s.a. square root filter or even hybrid methods.
- Extend the implementation of ensemble filters for other regions of the near Earth space