A numerical and experimental study of Galilei-transformed nonlinear wave groups

Yuchen He[1]**,** Guillaume Ducrozet[2], Norbert Hoffmann[3,4], John M. Dudley[5], and Amin Chabchoub[1,6]

•[1]School of Civil Engineering, The University of Sydney, Sydney, Autralia
•[2]LHEEA, École Centrale Nantes, UMR CNRS No. 6598, Nantes, France
•[3]Dynamics Group, Hamburg University of Technology, Hamburg, Germany
•[4]Department of Mechanical Engineering, Imperial College London, London, United Kingdom
•[5]Institut FEMTO-ST, UMR 6174 CNRS-Université de Franche-Comté, Besançon,France
•[6]Disaster Prevention Research Institute, Kyoto University, Kyoto, Japan

Theory & Background

- Nonlinear Schrödinger equation (NLSE)

$$i\left(\psi_x + \frac{1}{c_g}\psi_t\right) + \delta\psi_{tt} + \nu|\psi|^2\psi = 0$$

$$c_g = \frac{\omega}{2k}, \delta = -\frac{1}{g}, \nu = -k^3, \omega = \sqrt{gk}$$

V. E. Zakharov, Stability of periodic waves of finite amplitude on the surface of a deep fluid. *Journal of Applied Mechanics and Technical Physics 9*(2), 190-194 (1968).

- Complex wave envelope ψ

$$\boldsymbol{\psi}(x,t) \in \mathbb{C}$$

- Wave elevation η

$$\eta(x,t) = \mathrm{Re}\left(\boldsymbol{\psi}(x,t)e^{i(kx-\omega t)}\right) \in \mathbb{R}$$

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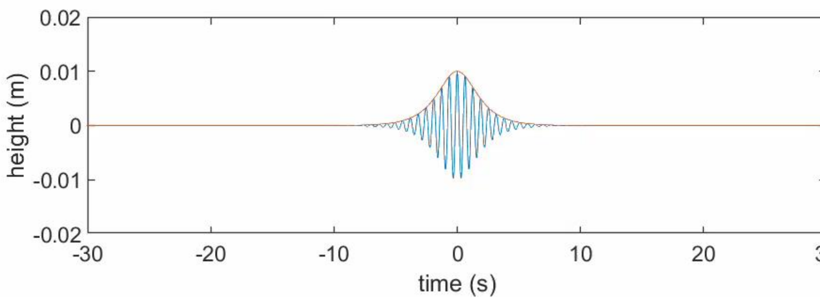
- Sech envelope soliton – **stable**

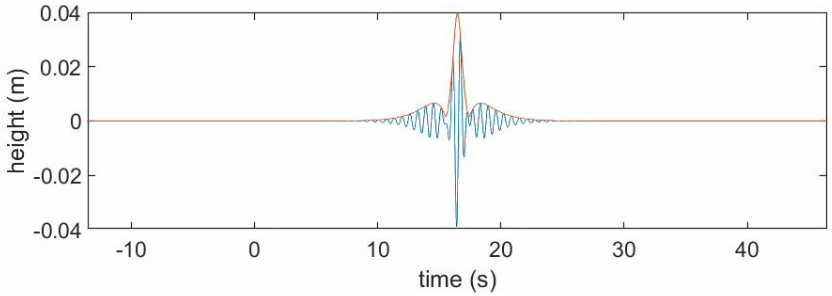
$$\psi_S(x,t) = a\ \operatorname{sech}\left(-\sqrt{2}\alpha k c_g\left(t-\frac{x}{c_g}\right)\right)\exp\left(-\frac{i\alpha^2 kx}{2}\right)$$

H. C. Yuen and B. M. Lake, Nonlinear deep water waves: theory and experiment. *The Physics of Fluids 18*(8), 956-960 (1975).
A. Slunyaev, G. F. Clauss, M. Klein and M. Onorato, Simulations and experiments of short intense envelope solitons of surface water waves. *Physics of Fluids 25*(6), 067105 (2013).



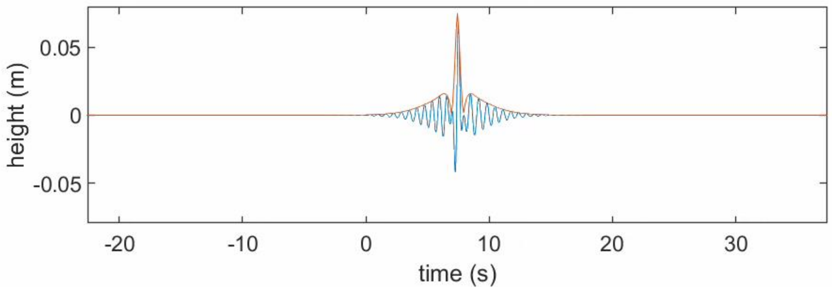
N=1

- Multi-soliton – **breathe**



N=2

$$\psi_{MS}(x=0,t) = N\,\psi_S(x=0,t), N \in \mathbb{N} \text{ (initial condition)}$$



N=3

L. F. Mollenauer, R. H. Stolen and J. P. Gordon, Experimental observation of picosecond pulse narrowing and solitons in optical fibers. *Physical Review Letters 45*, 1095 (1980).
A. Chabchoub, N. Hoffmann, M. Onorato, G. Genty, J. M. Dudley and N. Akhmediev, Hydrodynamic supercontinuum. *Physical Review Letters 111*, 054104 (2013).

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Research gap

Fundamental properties of NLSE

- › NLSE amplitude scaling transformation
- › NLSE time reversal symmetry

  A. Chabchoub and M. Fink, Time-reversal generation of rogue waves, Physical Review Letters 112, 124101 (2014).
- › NLSE Galilean symmetry

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- **Galilean transformation**

$$\psi_{\mathrm{G}}(x,t) = \psi\left(x, t - \frac{\sqrt{2}c\alpha}{4c_g}x\right)$$

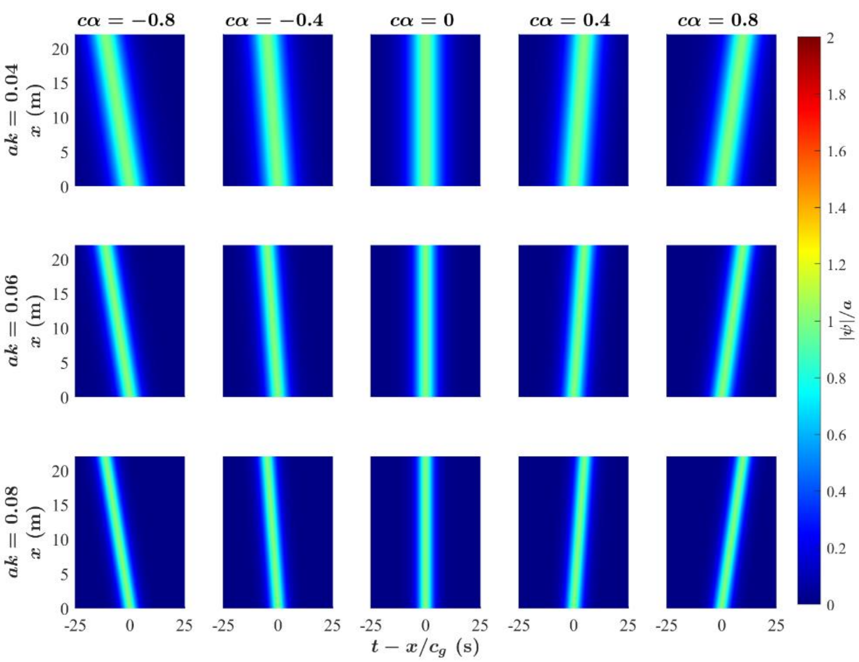
$$\exp\left(\frac{i\sqrt{2}c\alpha k(x - c_g t)}{2} + \frac{ic^2\alpha^2 kx}{8}\right)$$

- **Galilean-transformed soliton**

$$\psi_{\mathrm{GS}}(x,t) = a\ \operatorname{sech}\left(-\sqrt{2}\alpha k c_g\left(t - \frac{x}{c_g}\right) + \frac{c}{2}\alpha^2 kx\right)$$

$$\exp\left(-\frac{i\alpha^2 kx}{2}\right)\exp\left(\frac{i\sqrt{2}c\alpha k(x - c_g t)}{2} + \frac{ic^2\alpha^2 kx}{8}\right)$$

Theory (NLSE)

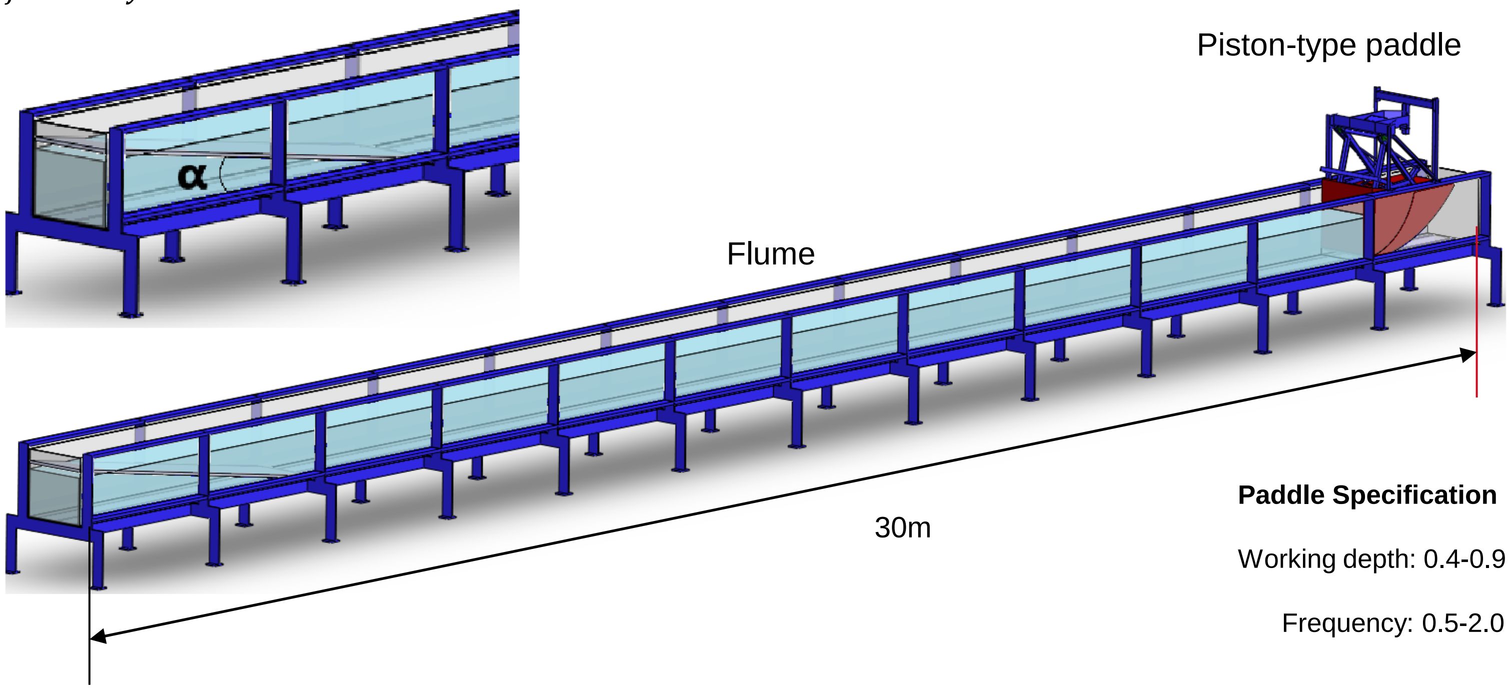


Experiments

Absorptive beach

$Reflectivity \sim 0$ if $\alpha \approx 8°$

Piston-type paddle



Paddle Specification

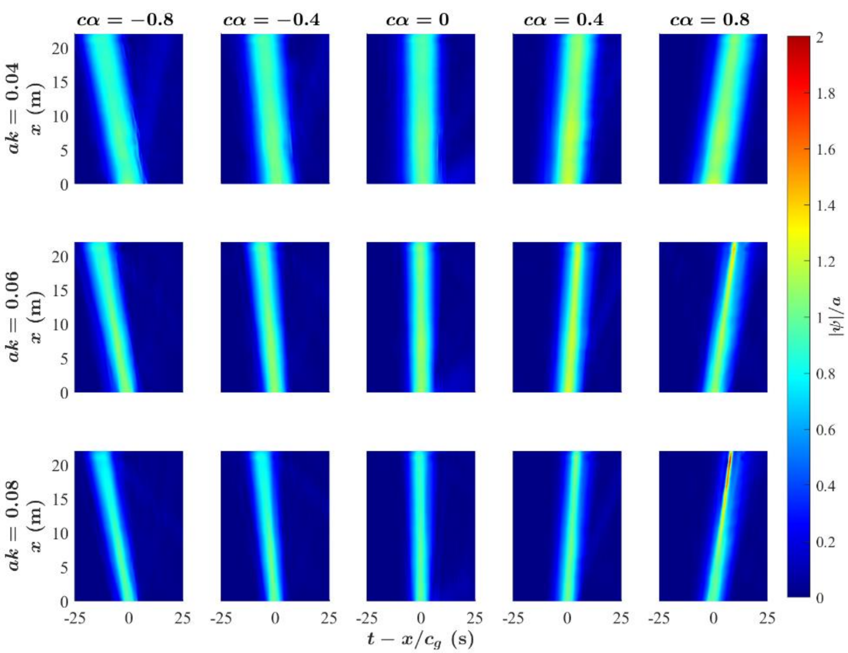
Working depth: 0.4-0.9 m

Frequency: 0.5-2.0 Hz

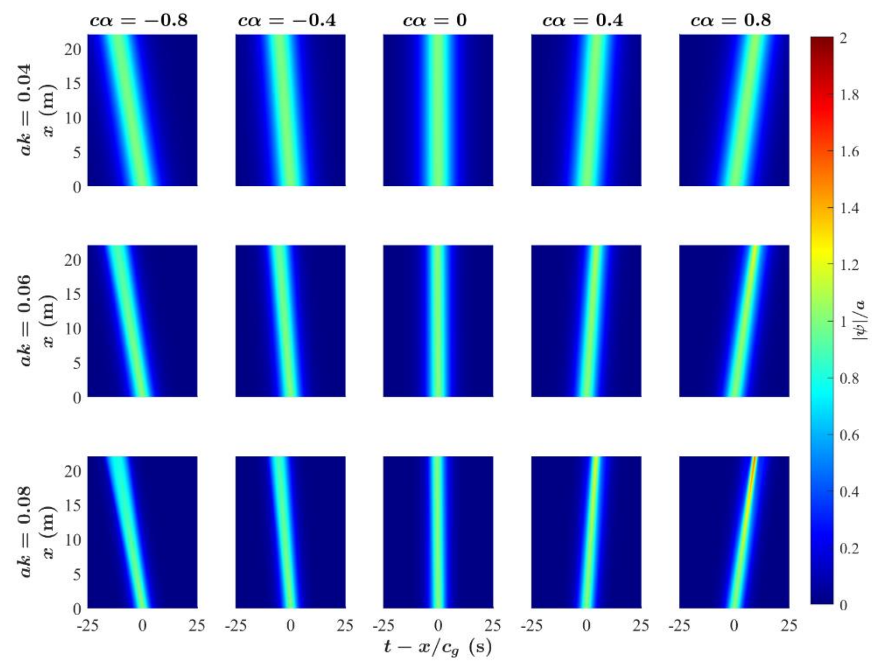
Working frequency (for original envelope soliton waves): 1.5 Hz

1. Higher-order NLSE modelling (MNLSE)

Experimental results



Higher-order model



Sharing is encouraged

E. Kit and L. Shemer, Spatial versions of the Zakharov and Dysthe evolution equations for deep-water gravity waves. Journal of Fluid Mechanics, 450, 201-205 (2002).

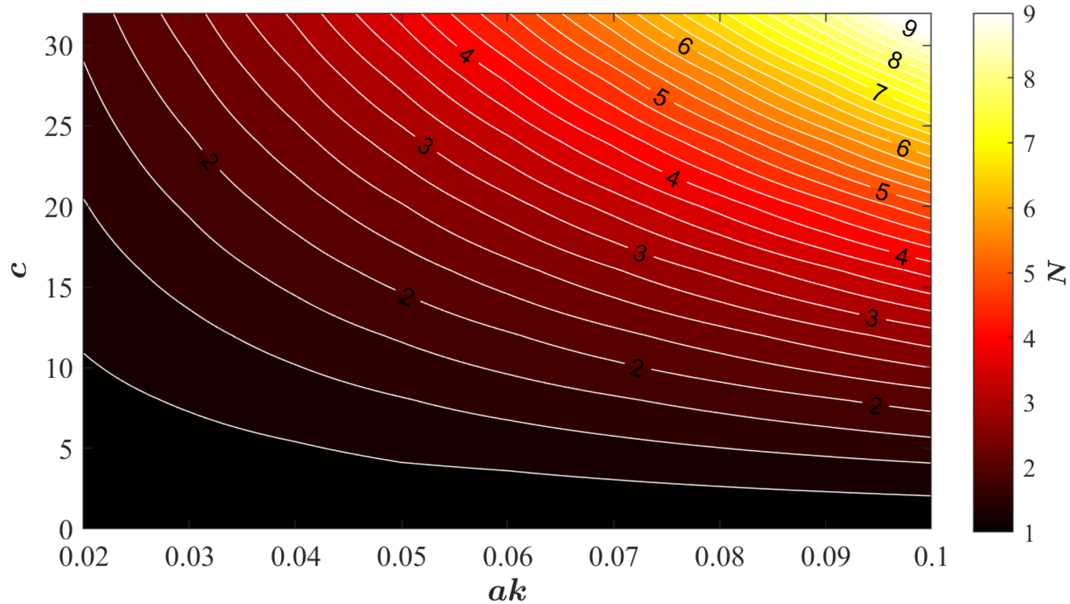
A. Goullet and W. Choi, A numerical and experimental study on the nonlinear evolution of long-crested irregular waves. *Physics of Fluids* 23, 016601 (2011).

2. Soliton order effect

- In terms of initial condition, we let

$$\psi_{\mathrm{G,sech}}(a,k,c,x=0,t) = R\psi_{\mathrm{sech}}(a',k',x=0,t),$$

where R is the order factor, $a' = \frac{a}{R}$.

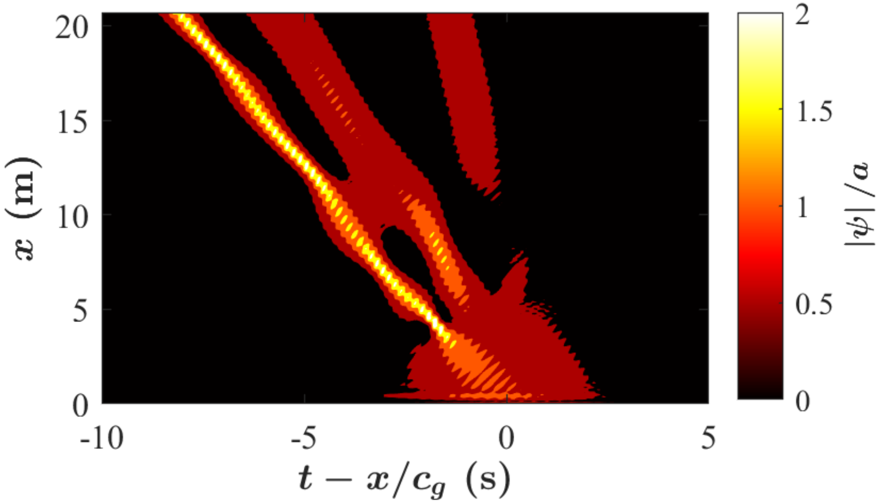


- It was derived analytically that

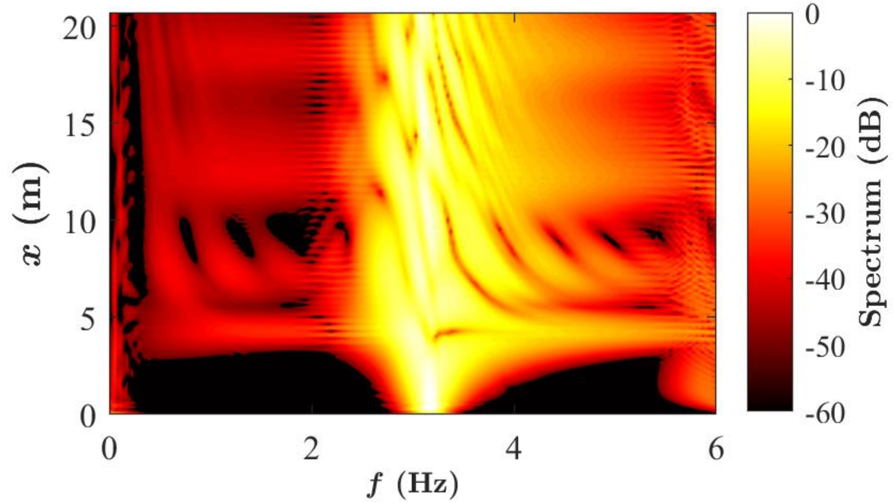
$$R = \left(1 + \frac{\sqrt{2}}{4}cak\right)^3$$

HOSM simulation of Galilean envelope soliton (Multi-soliton of order 4)

Envelope evolution



Spectrum evolution



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- **Summary**
  - At $\mathcal{O}(\epsilon^3)$ level the Galilean envelope solitons are steady;
  - Experiments do not support the steadiness of the solution;
  - The MNLSE shows good agreements with the experiments;
  - Connection between Galilean-transformed soliton and multi-soliton dynamics;
  - Soliton fission and supercontinuum as a result.
- Results have been published on Physica D:
  https://www.sciencedirect.com/science/article/pii/S0167278922001117

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