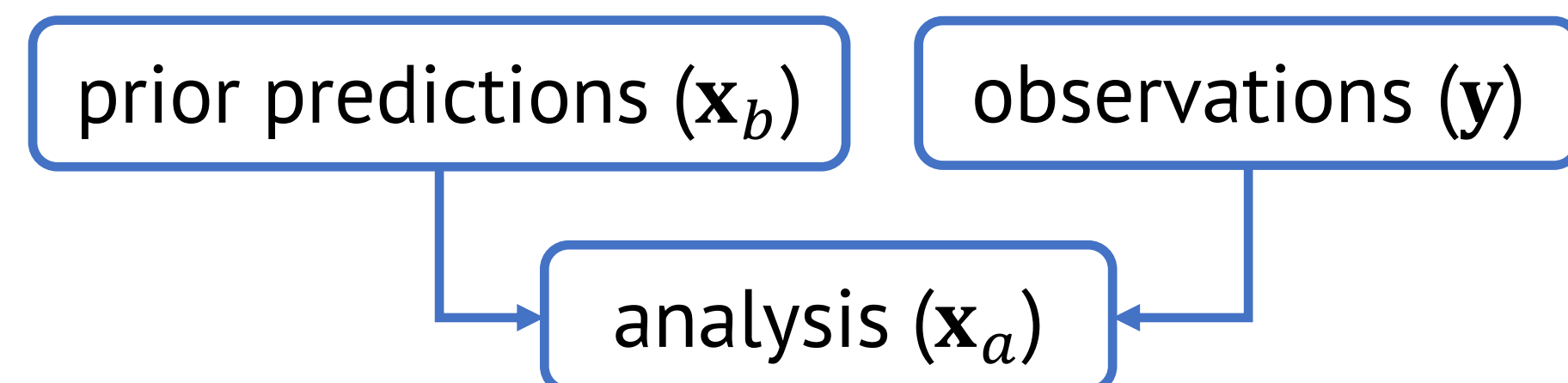


Background

Data Assimilation



Variational Methods: $\mathbf{x}_a = \arg \max_{\mathbf{x}} p(\mathbf{x}|\mathbf{x}_b, \mathbf{y})$

$$\mathbf{x}_a = \arg \min_{\mathbf{x}} (\mathcal{L}_o(\mathbf{x}, \mathbf{y}) + \mathcal{L}_b(\mathbf{x}, \mathbf{x}_b))$$

observation term background term

$$\mathcal{L}_o(\mathbf{x}, \mathbf{y}) = -\log p(\mathbf{y}|\mathbf{x}) \quad \mathcal{L}_b(\mathbf{x}, \mathbf{x}_b) = -\log p(\mathbf{x}|\mathbf{x}_b)$$

$\mathbf{y}|\mathbf{x} \sim \mathcal{N}(\mathcal{H}(\mathbf{x}), \mathbf{R})$ High dimensional
Complicated

$$\mathcal{L}_o(\mathbf{x}, \mathbf{y}) = \frac{1}{2} (\mathbf{y} - \mathcal{H}(\mathbf{x}))^T \mathbf{R}^{-1} (\mathbf{y} - \mathcal{H}(\mathbf{x}))$$

Traditional Algorithm (3DVar)

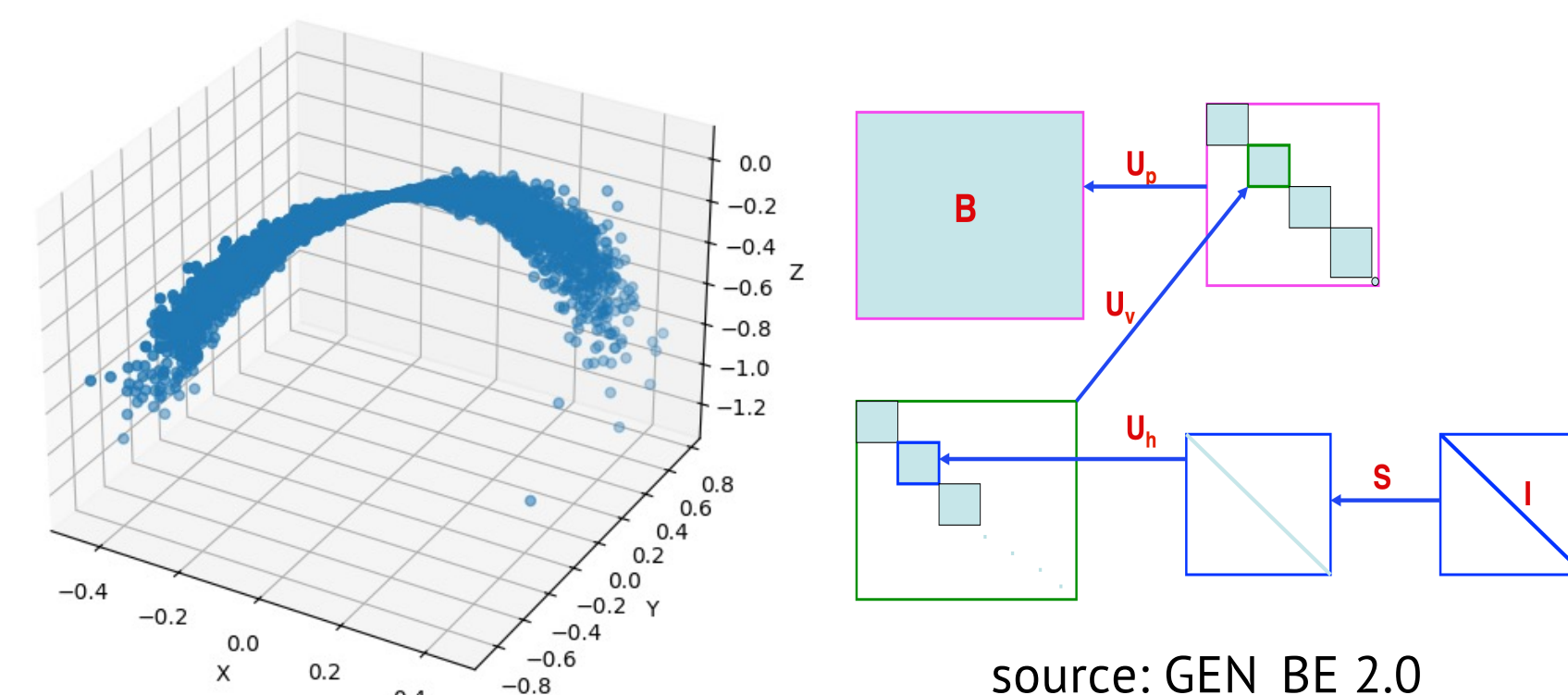
Assumption 1: $\mathbf{x}|\mathbf{x}_b \sim \mathcal{N}(\mathbf{x}_b, \mathbf{B})$

Assumption 2: $\mathbf{B} = \mathbf{U}\mathbf{U}^T, \mathbf{U} = \mathbf{U}_p \mathbf{U}_v \mathbf{U}_h \mathbf{S}$

VAE-Var

Purpose: improve the formulation of the background term with the help of neural networks.

Motivation



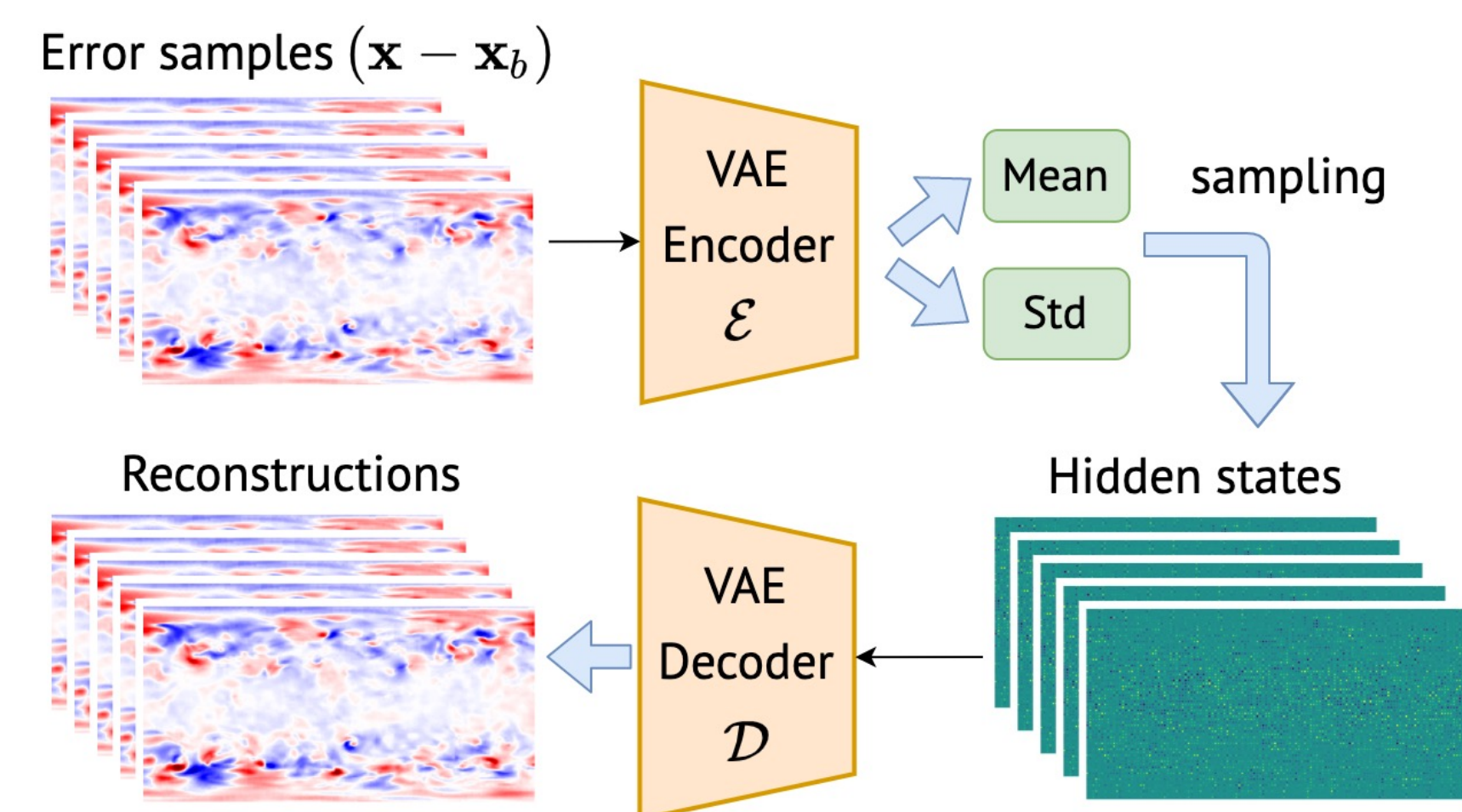
Gaussian assumption may not be correct.

Heavily rely on expert knowledge

Learning $p(\mathbf{x}|\mathbf{x}_b)$ with a generative neural network

Why VAE (instead of diffusion)?
Straightforward formulation, easy to train.

VAE Approximation



$$\log p(\mathbf{x}|\mathbf{x}_b) = \log p_{\delta}(\mathbf{x} - \mathbf{x}_b)$$

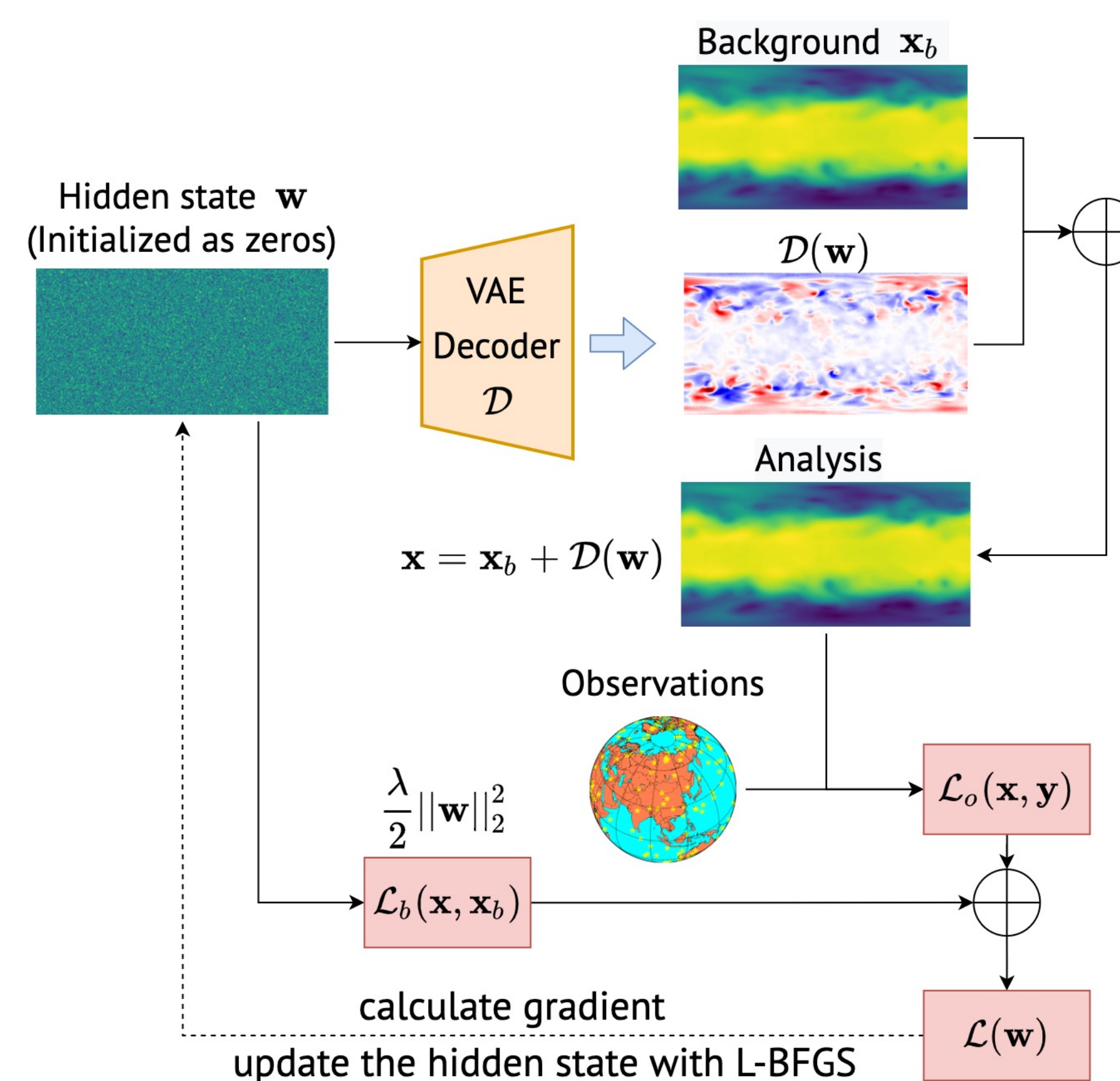
$$= \log \int p(\mathbf{x} - \mathbf{x}_b | \mathbf{z}) p(\mathbf{z}) d\mathbf{z}$$

$$= \log \int \exp\left(-\frac{\|\mathbf{x} - \mathbf{x}_b - \mathcal{D}(\mathbf{z})\|_2^2}{2\sigma^2}\right) \exp\left(-\frac{\|\mathbf{z}\|_2^2}{2}\right) d\mathbf{z}$$

Approximation: let $\mathbf{x} = \mathbf{x}_b + \mathcal{D}(\mathbf{w})$, then

$$-\log p_{\delta}(\mathcal{D}(\mathbf{w})) \leq \frac{\lambda}{2} \|\mathbf{w}\|_2^2 + C$$

VAE-Var Framework



Results

Dynamical System

WeatherBench (a global weather system).
Variables: **4 surface var. + 5 upper-air var.**
Horizontal resolution: **0.25° (1440×721)**
Vertical resolution: **13 layers**

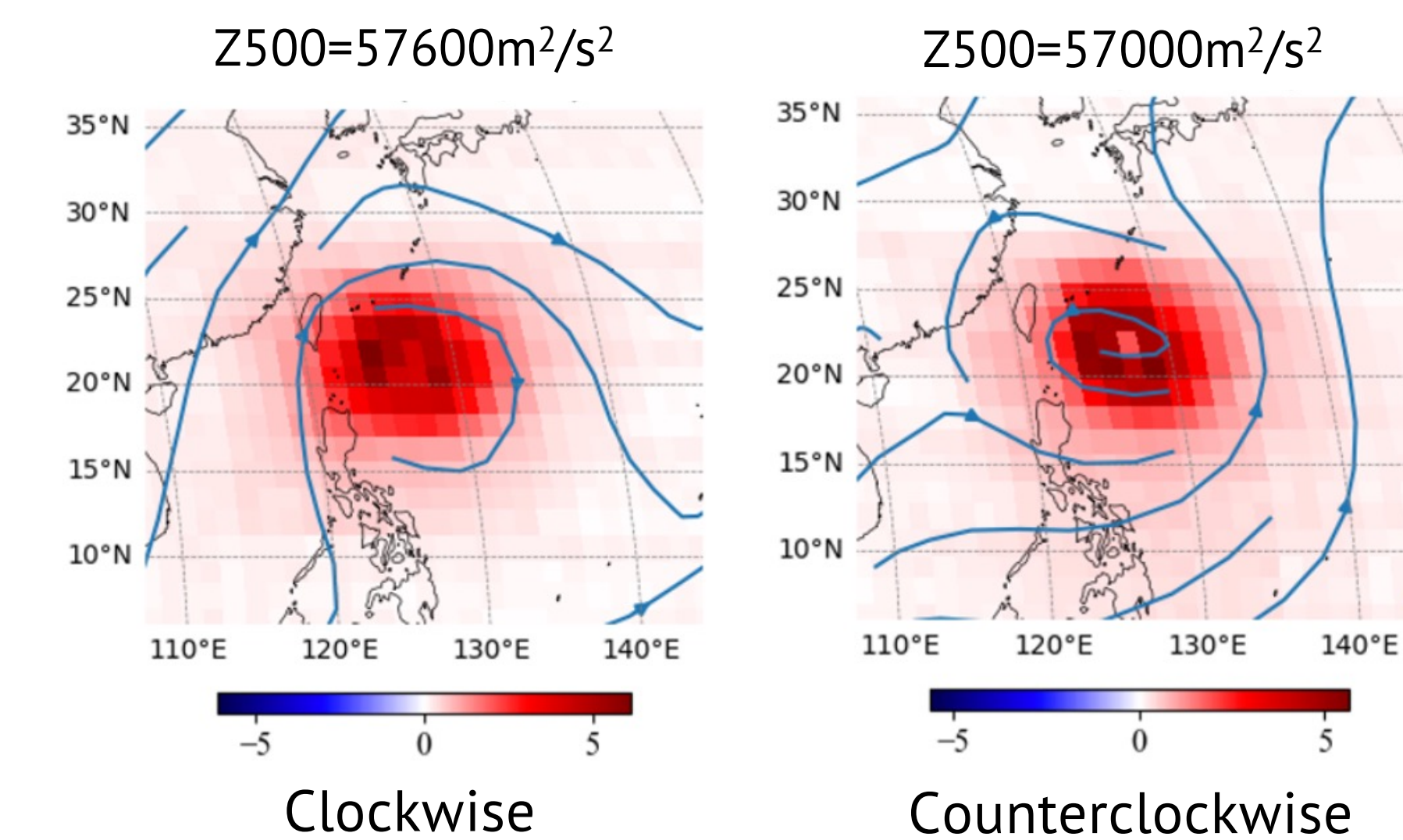
Forecasting Model

FengWu (a prominent global AI weather forecasting model)

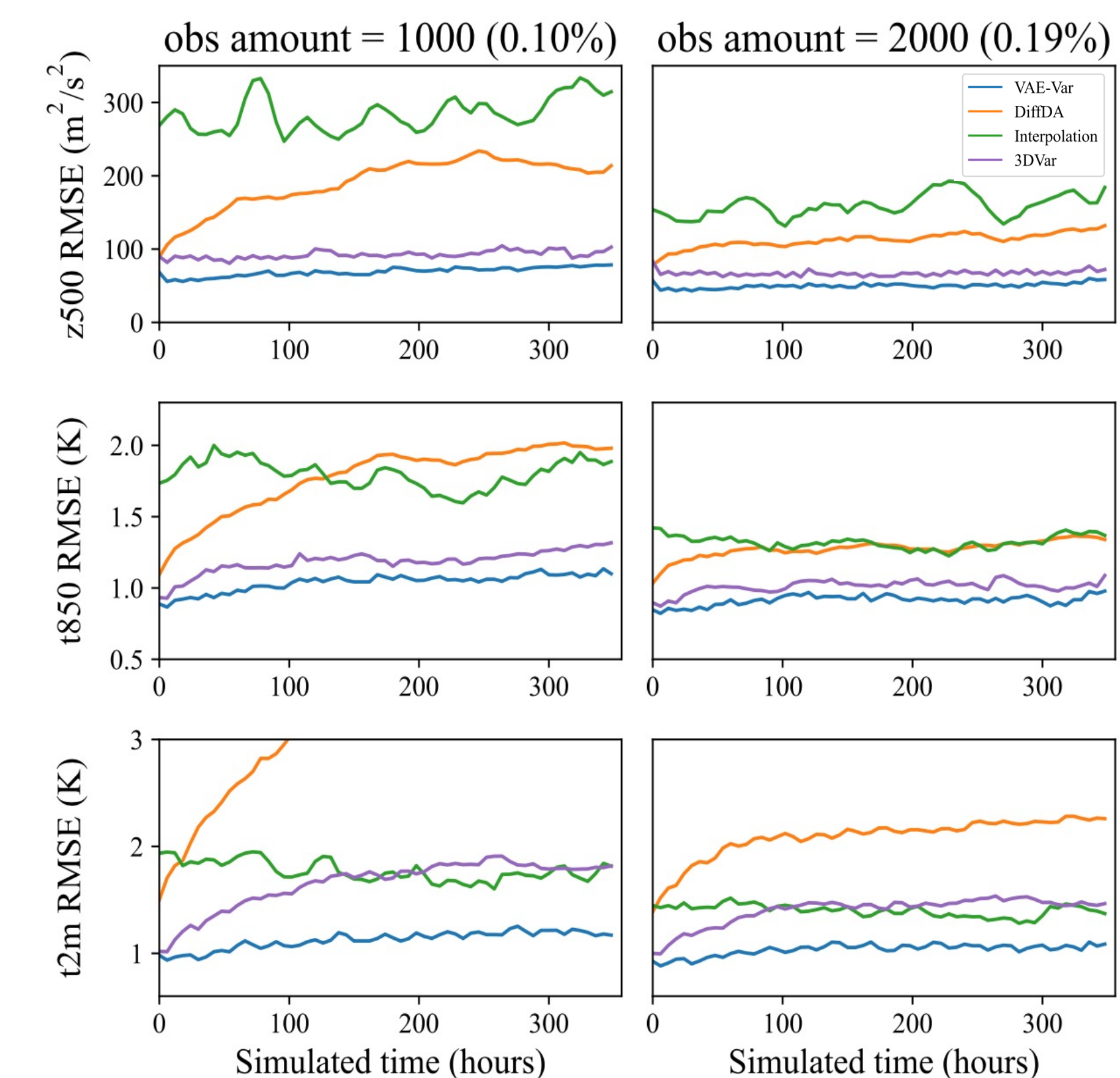
Single-Point Observation

How does an observation of Z500 at 21°N, 129°E affect the estimate of wind fields?

Analysis increment of the wind fields



Cyclic Forecasting



VAE-Var outperforms both 3DVar and DiffDA, especially under sparse observational settings.