EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Towards a new method for estimating englacial attenuation

Leah Sophie Muhle, Guy Moss, Clara Henry, Rebecca Schlegel and Reinhard Drews

This presentation participates in OSPP

EGU

Outstanding Student & PhD candidate Presentation contest

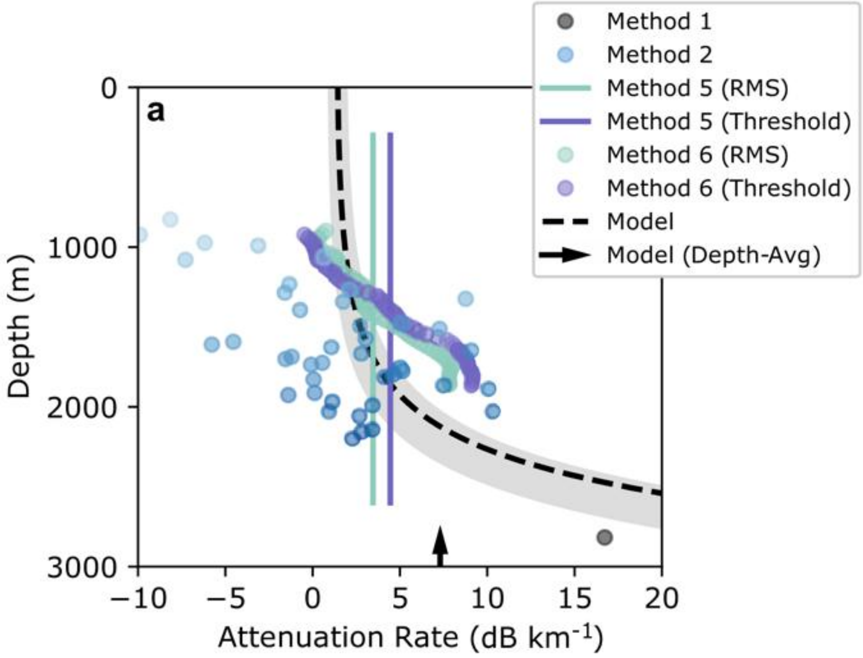
Tübingen AI Center

Emmy Noether-Programm

DFG Deutsche Forschungsgemeinschaft

HEINRICH BÖLL STIFTUNG

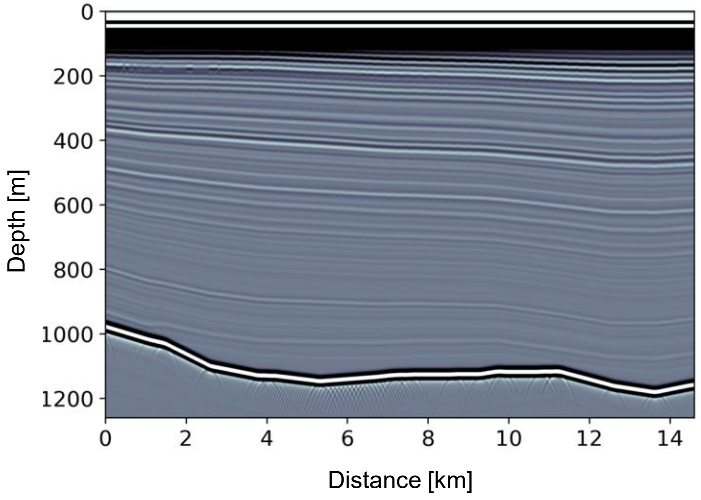
Precisely estimating englacial attenuation is still a problem …



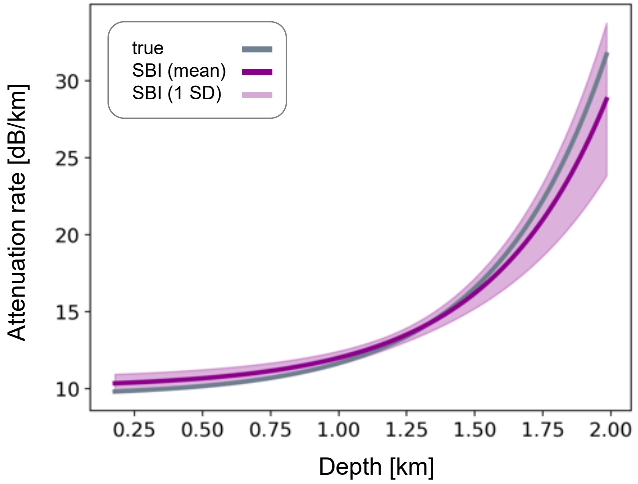
Hills et al., 2020

… so how do we improve it?

Use synthetic radar data for method validation

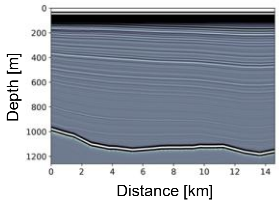


Get attenuation rate profiles with simulation-based inference

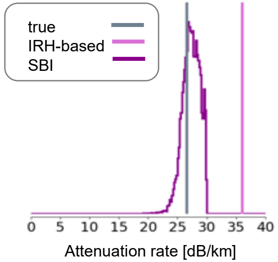


EBERHARD KARLS UNIVERSITÄT TÜBINGEN

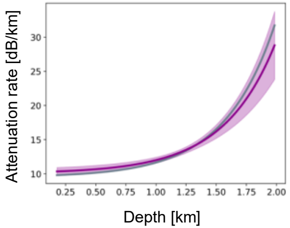
Overview



1. Validating inferred attenuation rates with synthetic radar data



2. Deriving depth-averaged attenuation rates with simulation-based inference



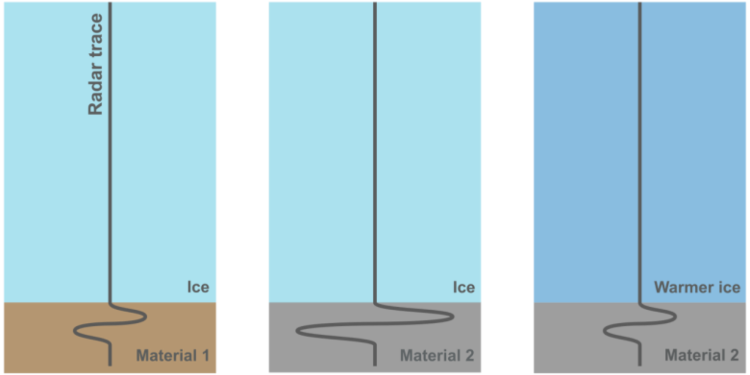
3. Deriving attenuation profiles with simulation-based inference

Why do we care about englacial radar attenuation?

$$N_a = \frac{10\log_{10} e}{1000\varepsilon_0\sqrt{\varepsilon'_{\text{ice}}}c}\sigma_\infty$$

$$\sigma_\infty = \sigma_{\text{pure}}\exp[\frac{E_{\text{pure}}}{k}(\frac{1}{T_r}-\frac{1}{T})] + \ldots$$

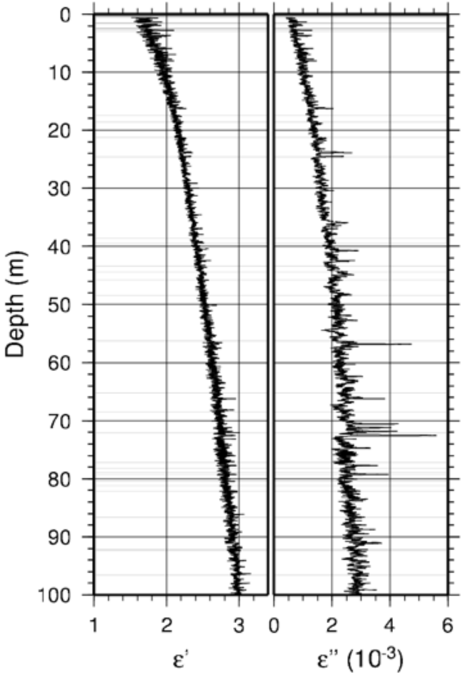
Proxy for **englacial temperature**



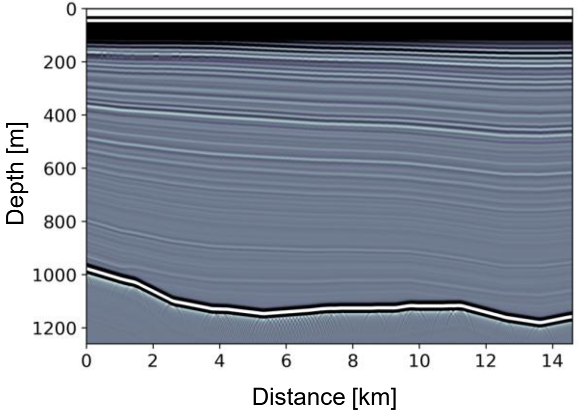
Correction factor for **basal reflectivity** estimates

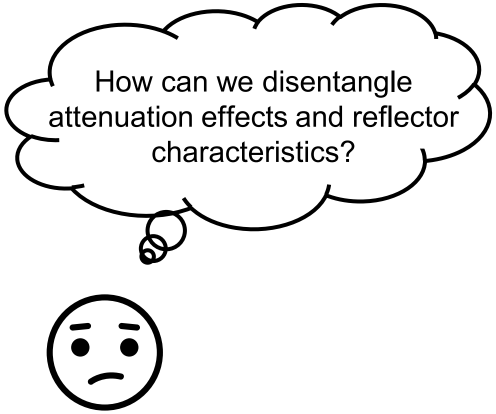
→ Better attenuation estimates could allow for **better constrained ice-flow models**

Varying dielectric contrasts in ice complicate inferring attenuation rates



Eisen et al., 2003

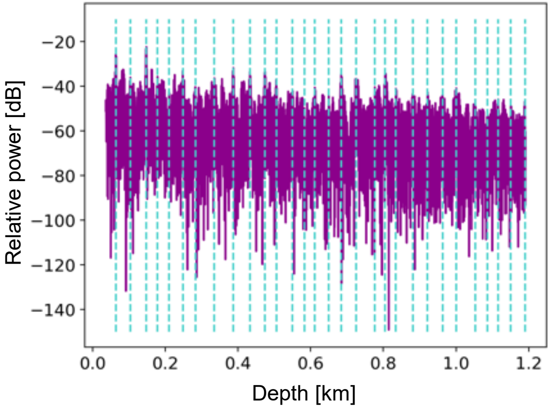




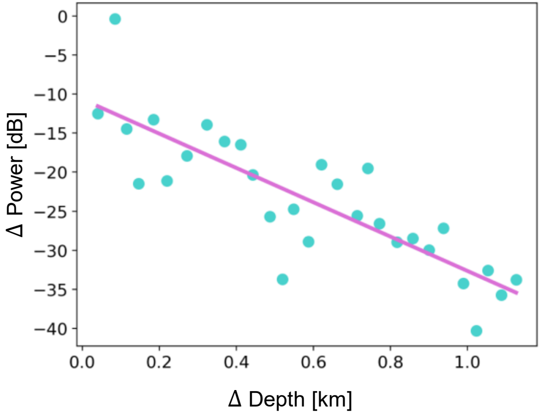
Validating depth-averaged attenuation rates derived from internal reflection horizons (IRHs)

How does the IRH-based method work?

1. Pick IRHs



2. Calculate power and depth difference to first IRH for each IRH



3. Derive depth-averaged attenuation rate $\langle N_a \rangle$ from slope

$$\boxed{\langle N_a \rangle = \text{-slope}/2}$$

Matsuoka et al., 2010, MacGregor et al., 2015

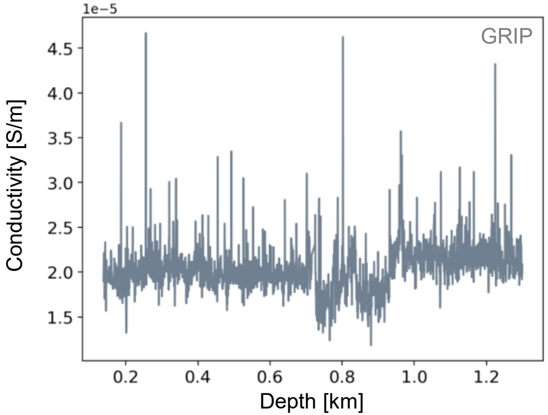
Limitations

- Assumes constant reflectivity of internal reflectors
- Only yields depth-averaged attenuation rates

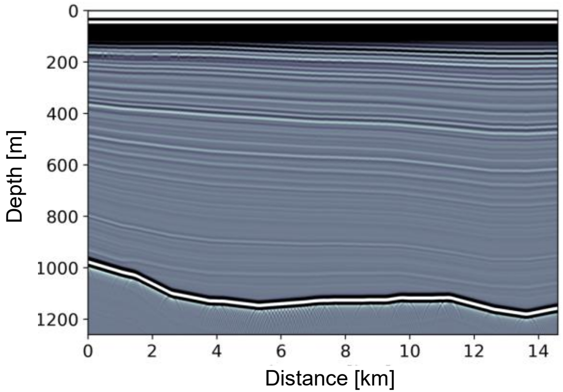
Generating synthetic radar data

Combine stratigraphy and temperature from an ice flow model …

… with a conductivity profile …



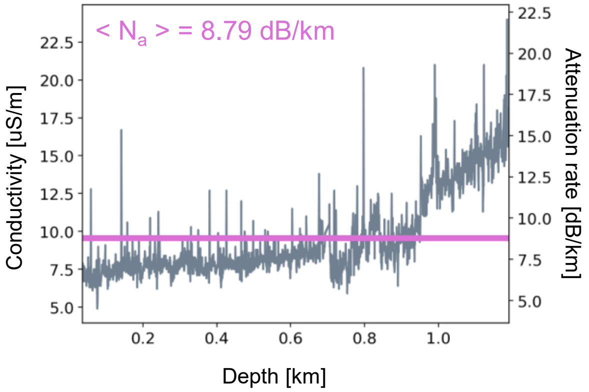
… and pass this to the radar forward model.



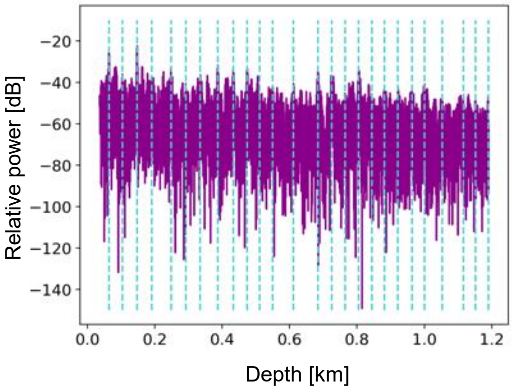
Click here to learn more about the radar model!

Validating the IRH-based method

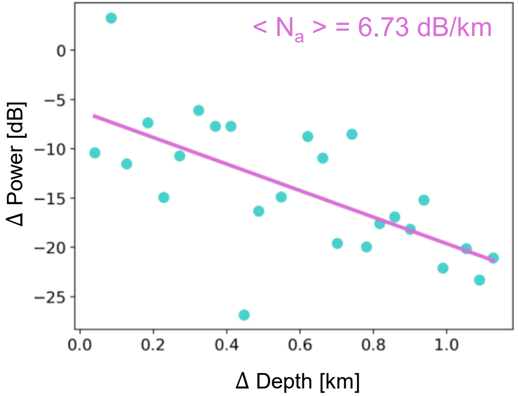
Underlying conductivity profile



Simulated radar trace



Derived < N_a >



→ IRH-based method roughly captures the underlying depth-averaged attenuation rate

Robustness of the IRH-based method

Trace length?

Number of IRHs?

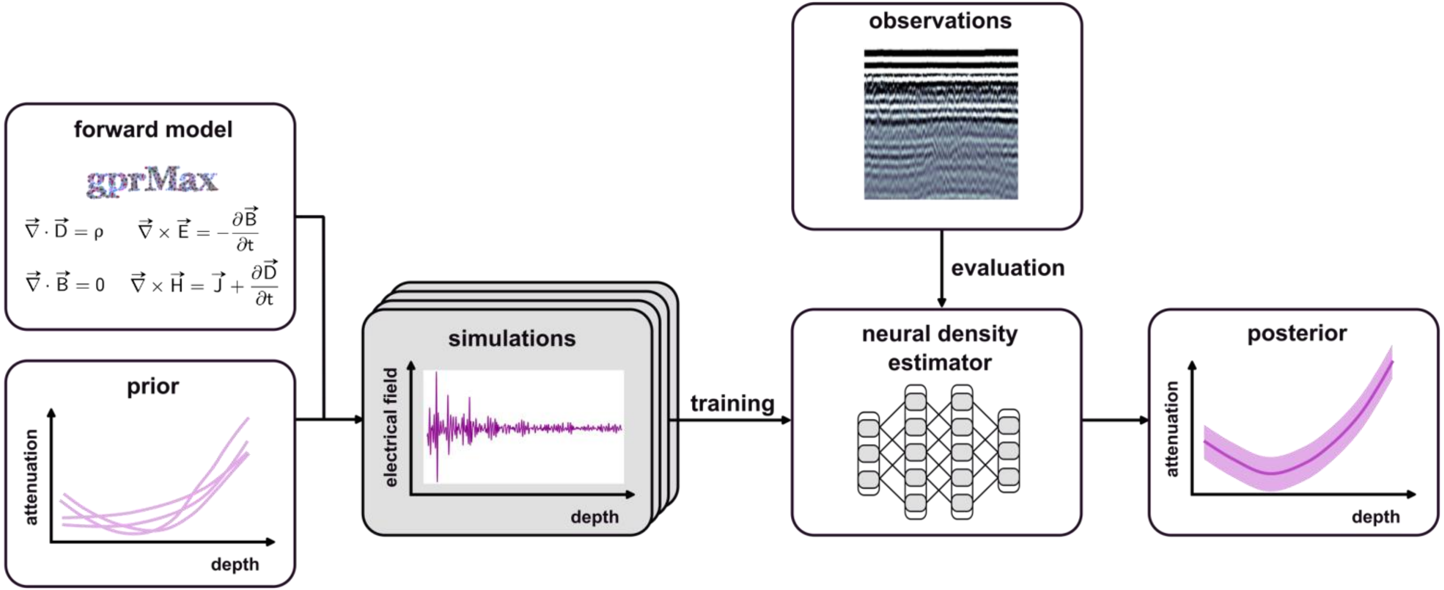
Potential influences on inferred < N_a >

Neglecting deep IRHs?

Choice of reference IRH?

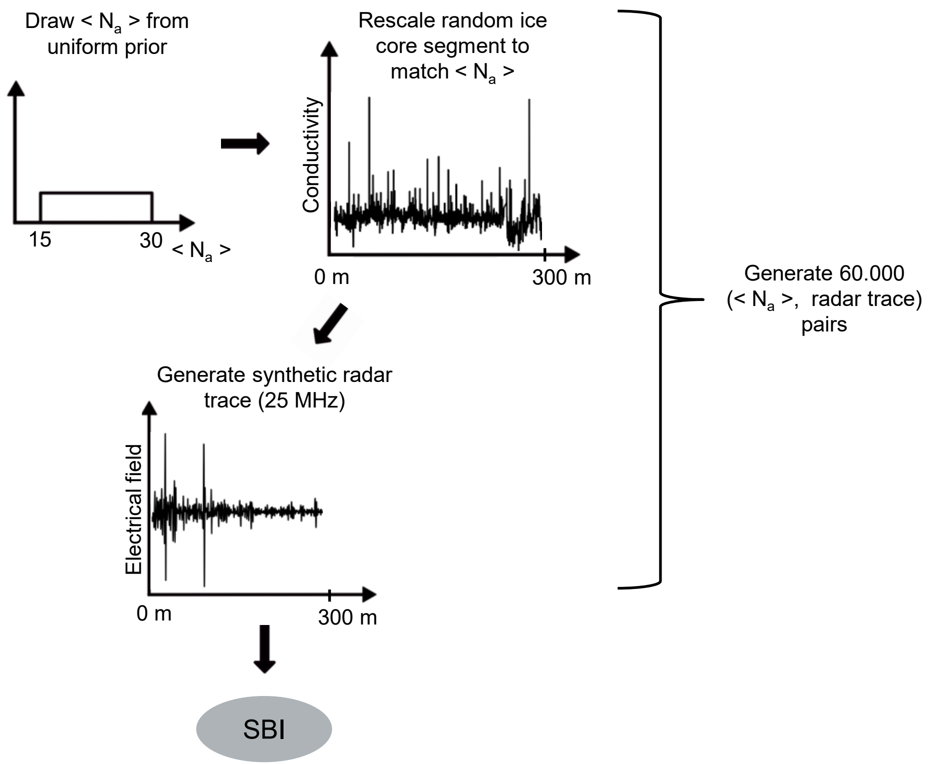
Inferring depth-averaged attenuation rates with simulation-based inference (SBI)

What is SBI?



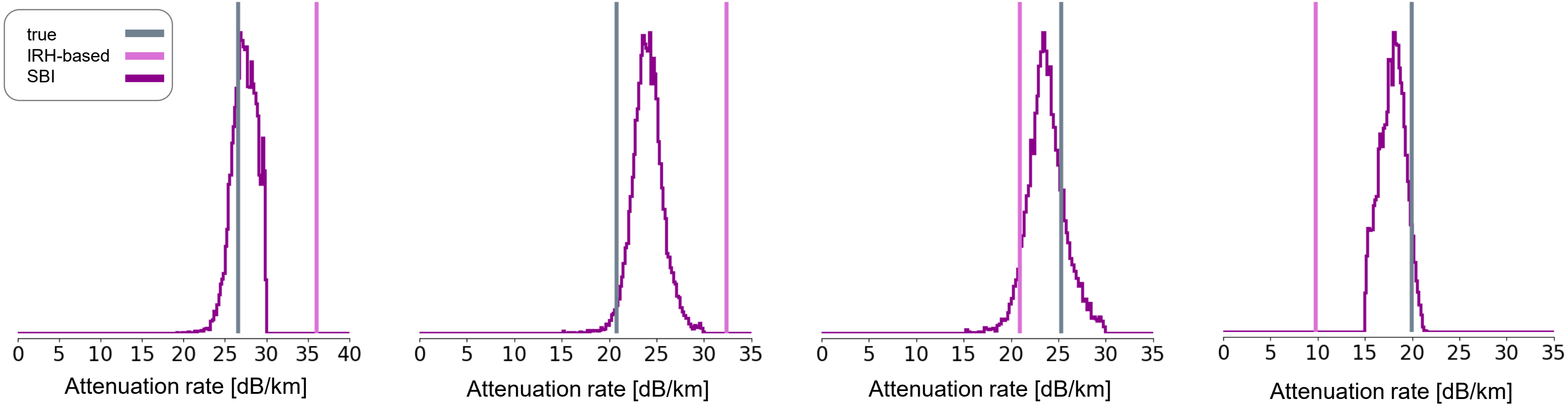
Click here to learn more about Bayesian inference!

Inferring < N_a > with SBI



SBI can predict < N_a > for shallow radar traces

Evaluated on synthetic radar traces



- Simple SBI setup can predict < N_a > even for shallow radar traces
- **SBI outperforms IRH-based approach** on shallow radar traces

First steps towards inferring attenuation profiles with SBI

How can we infer spatially varying profiles with SBI?

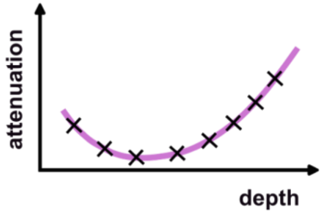
SBI can only infer a **limited number of discrete parameters**, which complicates its use for inferring spatially varying profiles.

Infer attenuation rates at fixed depths

Number of inference points defines the number of parameters

Limitations

- Limited flexibility due to discretization-dependence
- If number of inference points is too high, performance decreases

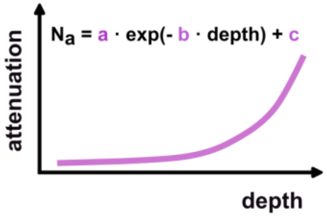


Infer function parameters

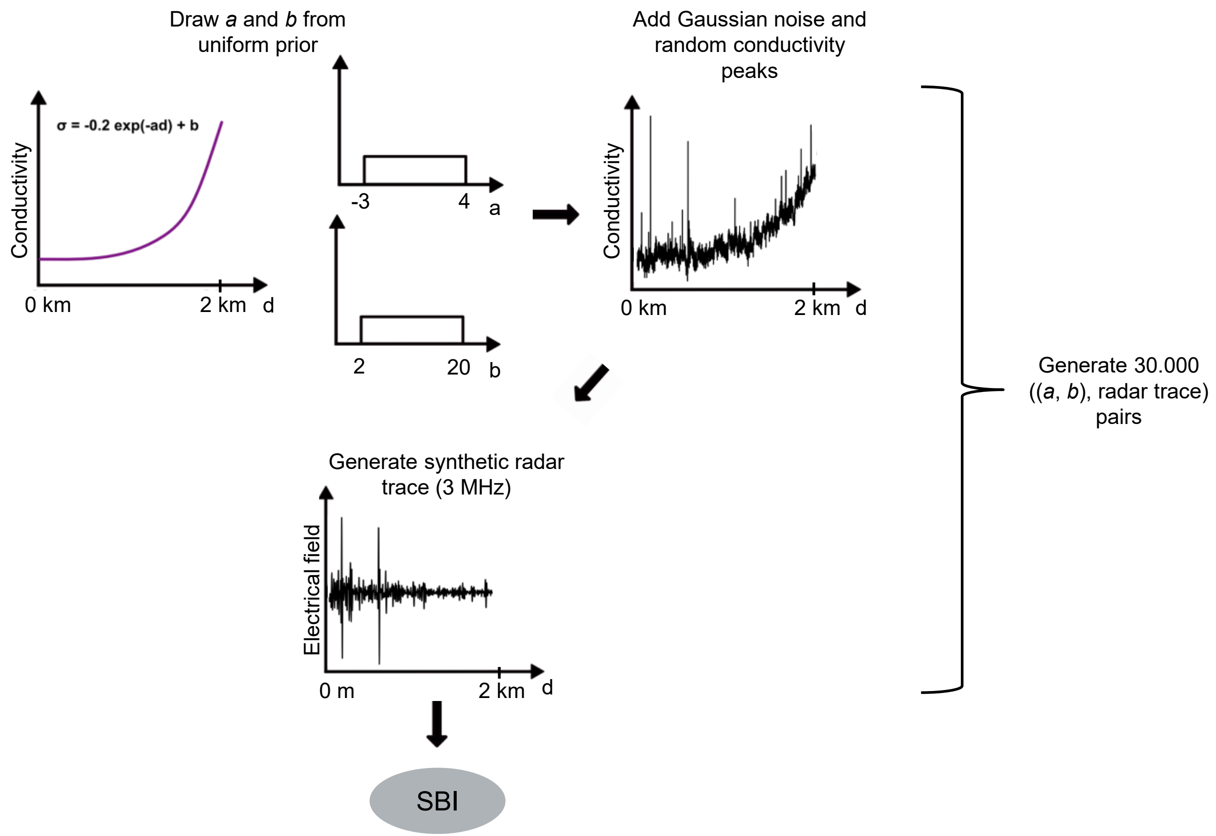
If functional shape of the profile is known, its parameters can be inferred

Limitations

- Limited transferability
- Highly sensitive towards small changes in parameters

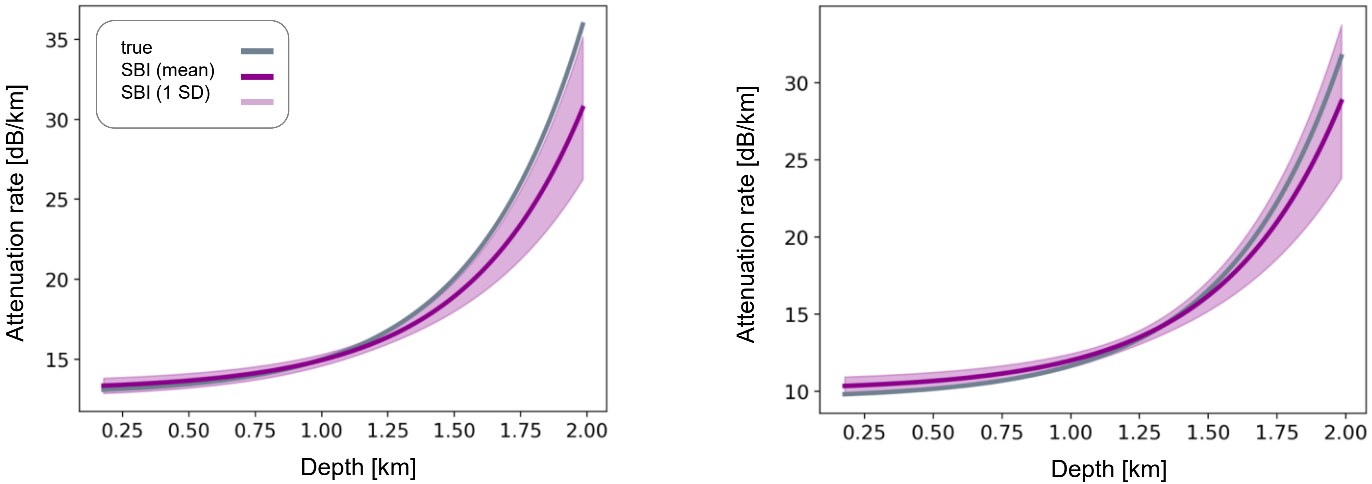


Inferring attenuation profiles with SBI



Basic SBI approach can roughly capture attenuation profiles

Evaluated on synthetic radar traces

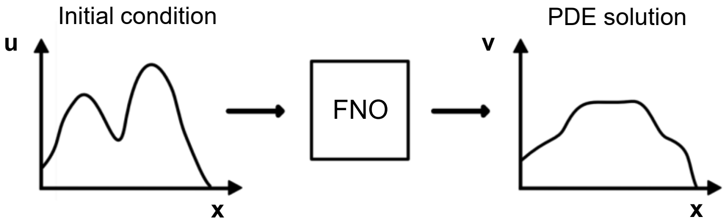


Limitation: Profiles are highly sensitive towards small changes in inferred parameters and not transferable to all thermal regimes.

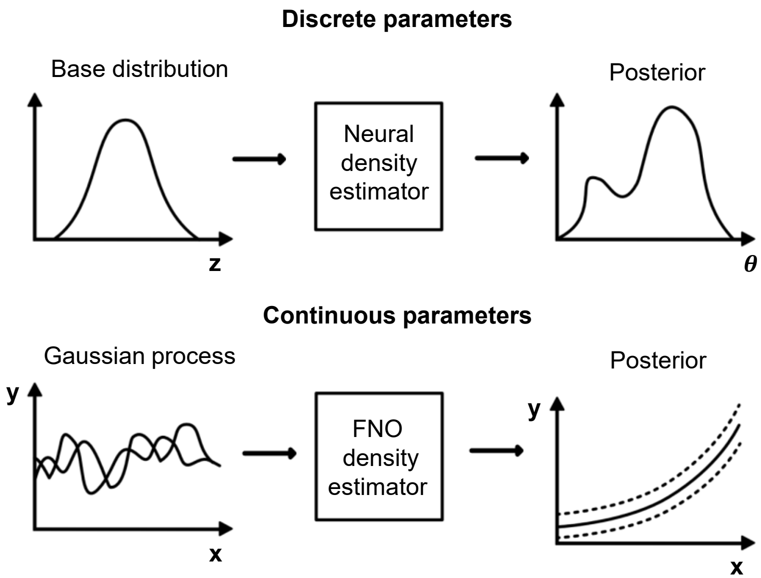
Possible solution: Fourier Neural Operators to infer continuous parameters

What are FNOs?

- Learn mappings between **function spaces**
- Work with **arbitrary discretizations**
- Can handle **highly dimensional data** by working in Fourier space with limited mode number

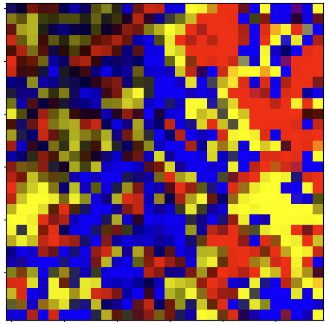


Probabilistic parameter inference

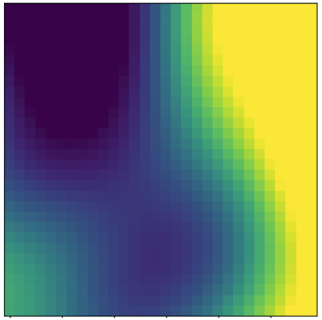


Continuous SBI: Preliminary results

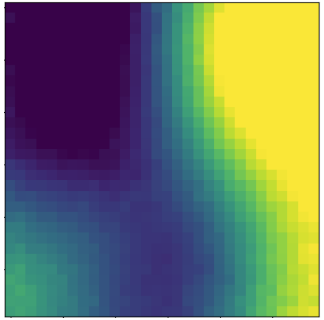
Observation



True parameters (900!)



Inferred parameters



Spatial density of three different predator populations

Spatially varying predation rate

- Only 10.000 simulations used in training
- For simple models, **continuous SBI can infer continuous parameters**

Joint project with Guy Moss

Next steps

1. Infer attenuation profiles with continuous SBI from synthetic radar data

2. Transfer from synthetic data to radar observations
    - Antenna-ice coupling?
    - Waveform?
    - Correct noise model? Noise library?

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Thanks to everyone involved!

Guy Moss

Clara Henry

Rebecca Schlegel

Reinhard Drews

If you have any questions or suggestions, please reach out.

Mail: leah-sophie.muhle@student.uni-tuebingen.de
Bluesky: @leahsmuhle.bsky.social

Emmy Noether-Programm DFG Deutsche Forschungsgemeinschaft

HEINRICH BÖLL STIFTUNG

Tübingen AI Center

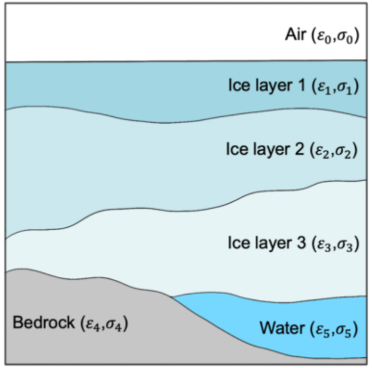
Generating synthetic radar data

gprMax

- Open-source software simulating electromagnetic wave propagation
- Solves Maxwell's equations numerically with finite-difference time-domain method for ground-penetrating radar
- GPU-acceleration possible

gprMax input

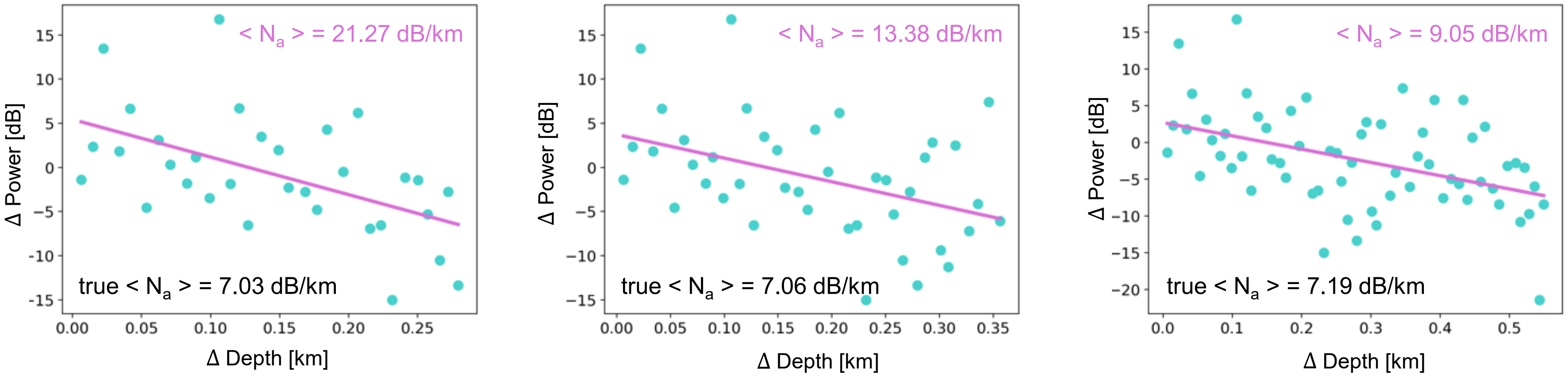
- Each combination of relative permittivity and conductivity constitutes a material (identified by an integer)
- Represent geometry as material array
- We always assume a constant relative permittivity of 3.17
- Cell resolution depends on frequency



Δy Δz

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1
2	2	1	1	1	1	1	1	1	2	2	1
2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	3	3	3	3
2	3	3	3	3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	3	3	3	3	3	3	3
4	4	4	4	4	4	4	5	5	5	5	5
4	4	4	4	4	4	4	4	4	5	5	5

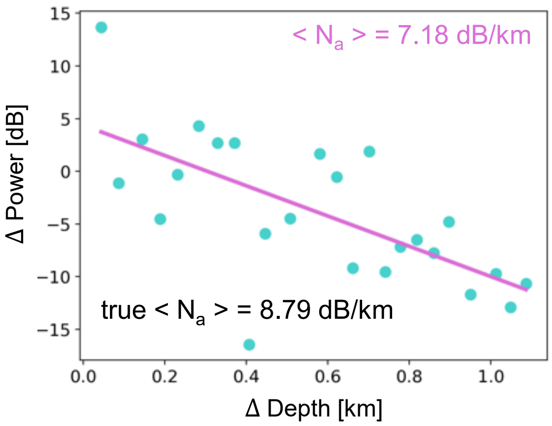
Trace length strongly influences inferred < N_a >



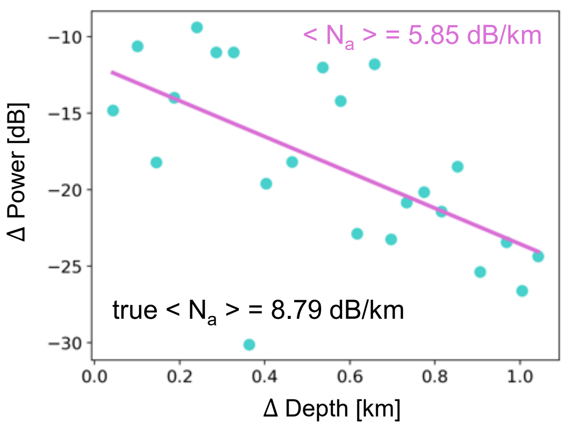
→ IRH-based method does not work well for shallow radar traces

Reference peak choice mildly influences inferred $< N_a >$

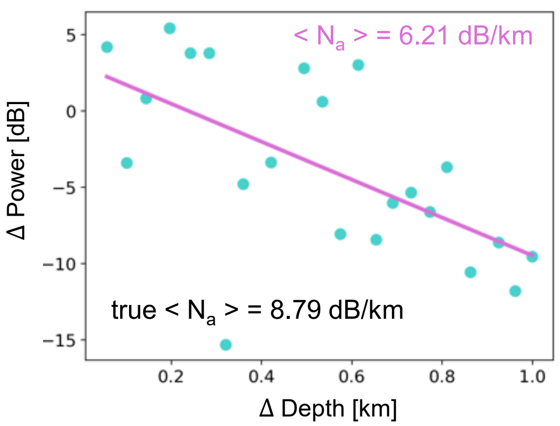
Second IRH as reference



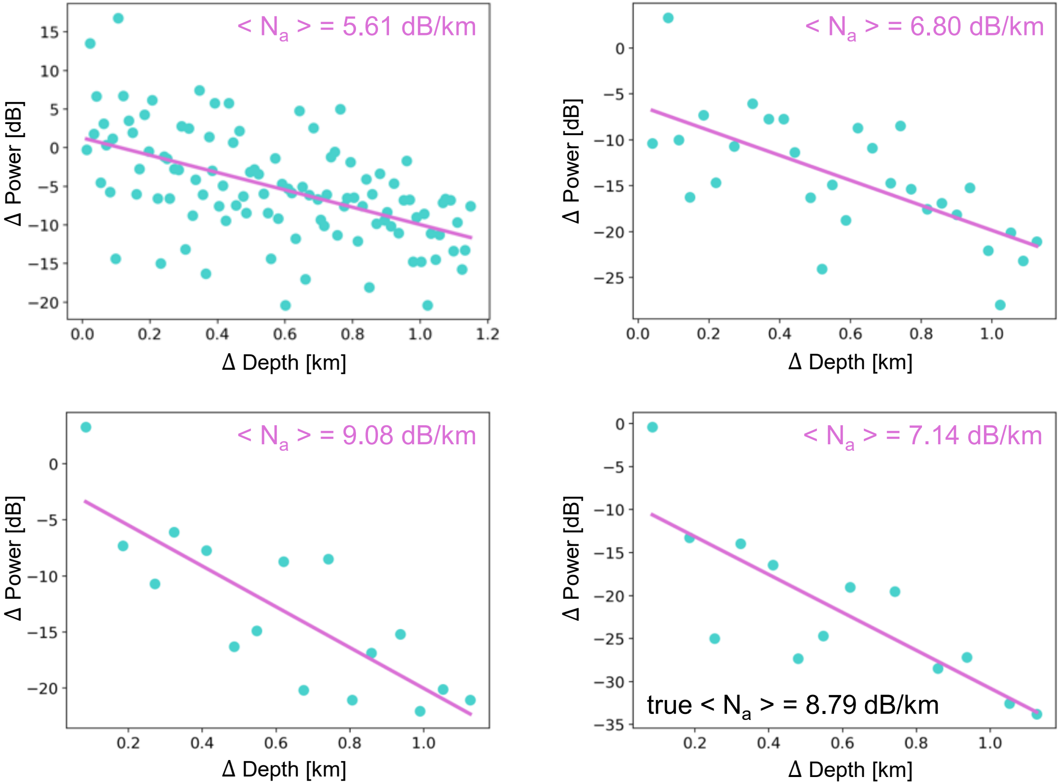
Third IRH as reference



Fourth IRH as reference



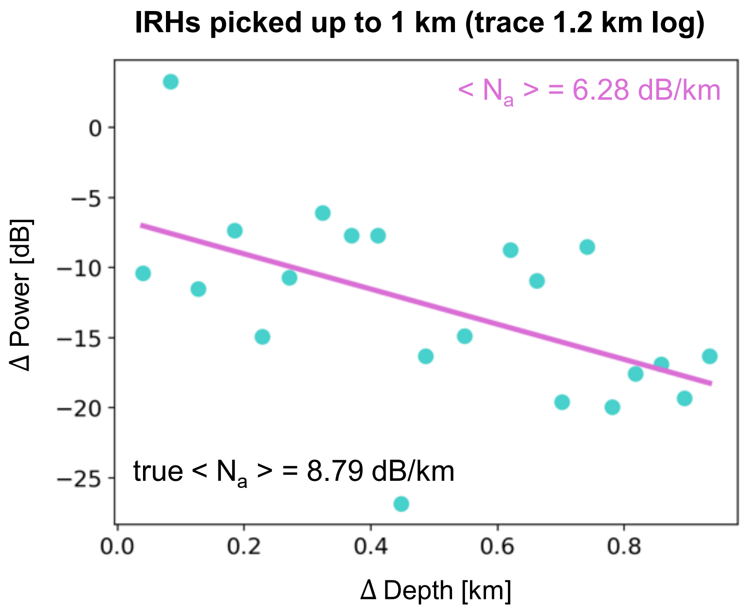
Number of picked IRHs mildly influences inferred $< N_a >$



→ Can too many picked IRHs distort the estimate?

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Neglecting deep IRHs might lead to underestimating $< N_a >$



Short intro to Bayesian inference

How likely are different radar observations given a particular attenuation rate?

What do we know about the attenuation rate before? How likely are different values of it?

likelihood

prior probability

$$p\,(\text{parameter} \mid \text{data}) = \frac{p\,(\text{data} \mid \text{parameter}) \cdot p\,(\text{parameter})}{p\,(\text{data})}$$

posterior probability

evidence

What are the probabilities of different attenuation rates given a particular radar observation?