An Analytic Element Method solution for multispecies reactive contaminant transport

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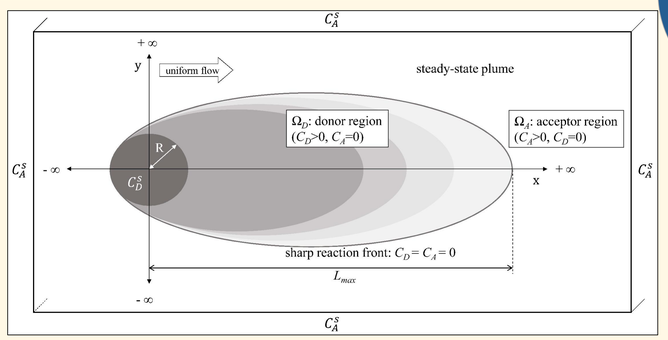
1 Motivation and Background

- Millions of sites still remain unassessed (EEA, 2021)*
- Efficient and effective models are still required
- Analytic Element Method (AEM) can provide such option
- AEM advantages include: grid-independence, exact solution, flexible domain and boundary conditions, domain orientation, complex source geometries

2 Model development



- Circular elements in a 2D horizontal domain with instantaneous reaction among binary reactants
- A coordinate transformation eliminates the anisotropy of the governing transport equation
- The isotropic equation is further transformed to the **modified Helmholtz equation**

$$v\frac{\partial C_D}{\partial x} - (\alpha_L v + D_D)\frac{\partial^2 C_D}{\partial x^2} - (\alpha_T v + D_D)\frac{\partial^2 C_D}{\partial y^2} = -r$$

$$v\frac{\partial C_A}{\partial x} - (\alpha_L v + D_A)\frac{\partial^2 C_A}{\partial x^2} - (\alpha_T v + D_A)\frac{\partial^2 C_A}{\partial y^2} = -\gamma r$$

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial Y^2} = \beta^2 F$$

- Concentration distribution is obtained from infinite series expansion of Mathieu functions:

$$F(\psi,\eta) = \sum_{n=0}^{\infty} a_n f(\psi,\eta)_{n,M} + b_n g(\psi,\eta)_{n,M}$$

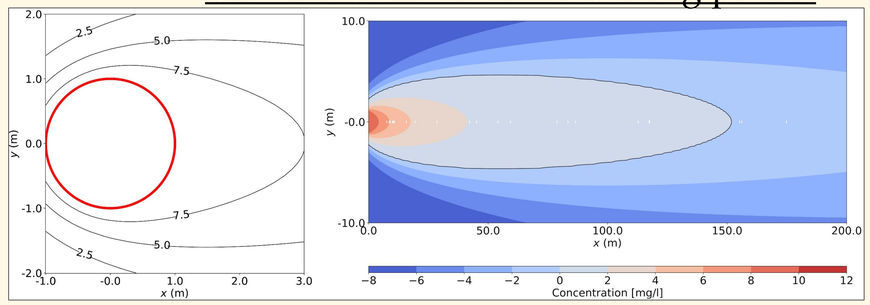
Advanced methods:

Method of images, Superposition

3 Results

- Maximum plume length can easily be obtained
- Sensitivity analysis: source concentration and stoichiometry most critical

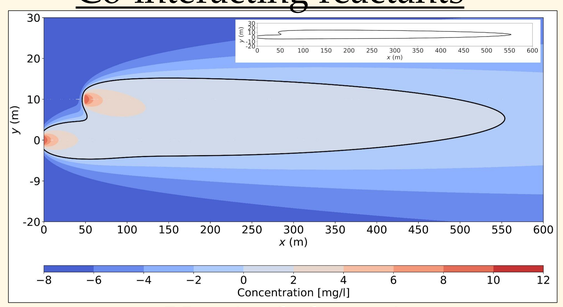
Circular element and resulting plume



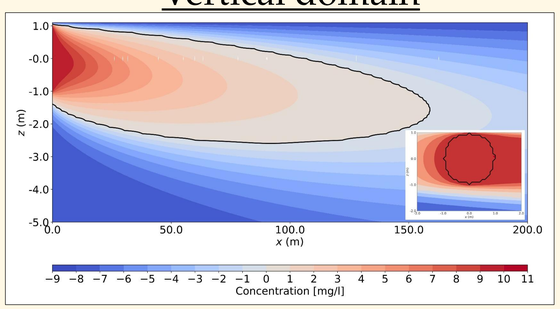
4 Application scenarios

- Several practical scenarios can be simulated
- Large scale modelling at minimal computational effort

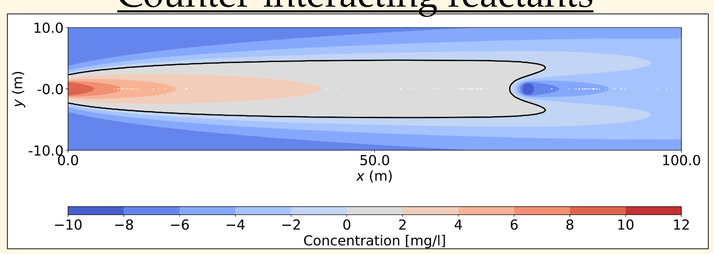
Co-interacting reactants



Vertical domain



Counter-interacting reactants



5 Conclusions and outlook

- The AEM can be used for efficient site assessment modelling
- Line elements can be developed for greater variety of applications
- Complex source geometries with and without discontinuities can be simulated

Find our abstract here

*Reference: https://www.eea.europa.eu/data-and-maps/indicators/progress-in-management-of-contaminated-sites-3/assessment

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