Adjoint-Based Sensitivity Analysis using Automatic Differentiation for Geoscientific Multiphysics Problems

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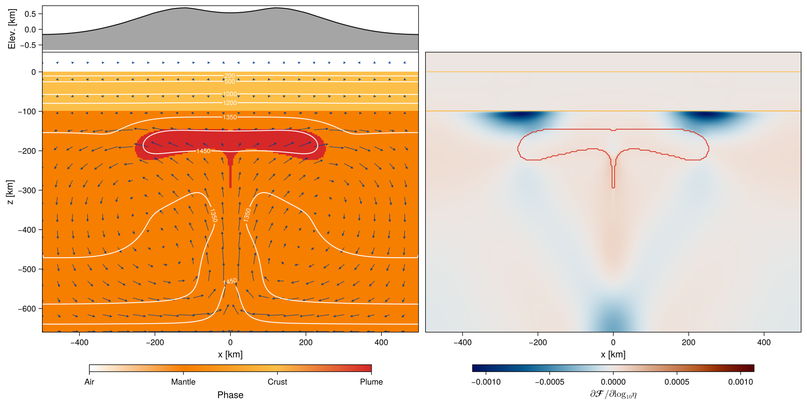
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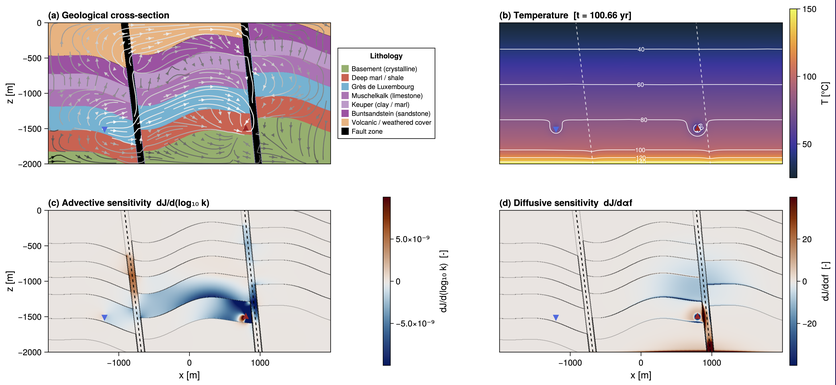
What are Sensitivity Kernels?

*Geophysical observations **at the surface** reflect the integrated effect of processes spanning the **entire domain**, but which parameters at depth actually control the signal?*



Sensitivity kernels** answer exactly this question. They map how much each **parameter** influences a given **observation.

Geothermal Heat Extraction



*What material parameters control **thermal power** at the production well?*

*Model of coupled **Darcy-flow** + **heat transport** in reprasentative central European sedimentary basin*

***Advective sensitivity:** Fault zones and sandstone aquifer between wells control heat extraction*

***Diffusive sensitivity:** conductive recharge controlled by thermal properties near conduction well*

Permeability** governs **large scale flow** routing, **thermal diffusivity** governs **local recharge.

***Adjoint variable** tells us where a new borehole would provide maximum information.*

How Does This Work?

Applying the Discretize-then-Optimize Method

Three Step Procedure:

1.) Solve forward problem & evaluate cost function

$$\mathbf{R}(\mathbf{u}, \mathbf{m}) = 0 \quad \Big| \quad F(\mathbf{u}) = \frac{1}{2} \sum_i (u_i - u_{i,\mathrm{obs}})^2$$

2.) Solve adjoint problem

$$\mathbf{J}^T \psi = \left(\frac{\partial F}{\partial \mathbf{u}} \right)^T \quad \text{where} \quad \mathbf{J} = \frac{\partial \mathbf{R}}{\partial \mathbf{u}}$$

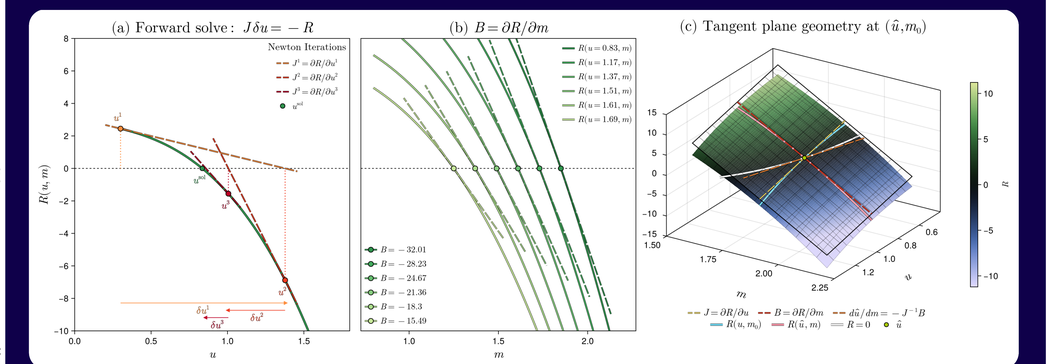
3.) Compute sensitivity kernels

$$\mathbf{K} = -\psi^T \mathbf{B} \quad \text{where} \quad \mathbf{B} = \frac{\partial \mathbf{R}}{\partial \mathbf{m}}$$

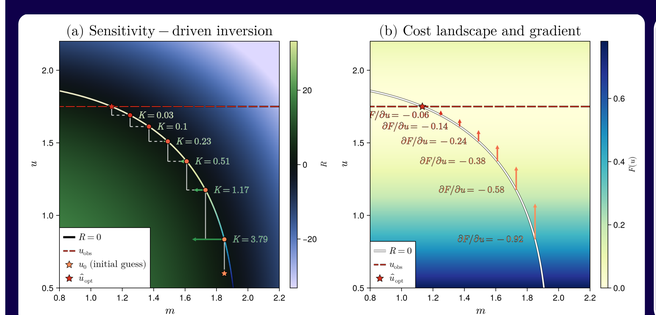
*This method can be used to evaluate gradients of a given cost function **F** w.r.t. the input parameters **m**. In our multiphysics problems, the cost function depends on the observations that we have. Check out the different examples to see how we solve different problems using the same method!*

Geometric Interpretation

To understand how the kernels are calculated, we visualize the assembled equations and gradients on the residual- and cost function-surfaces.



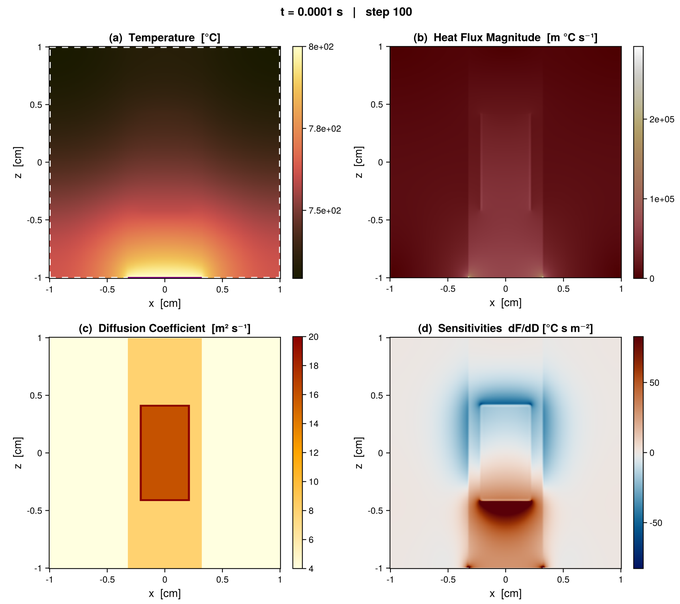
(a) Newton iterations to solve the forward problem.
(b) Gradient of Residual w.r.t. material parameters, $\mathbf{B}$. Note how nonlinearity changes $\mathbf{B}$ when solution changes
(c) Tangent plane that describes sensitivity calculation in the case that F = u



(a) R=0 contour colored by cost function value and ∂F/∂u.

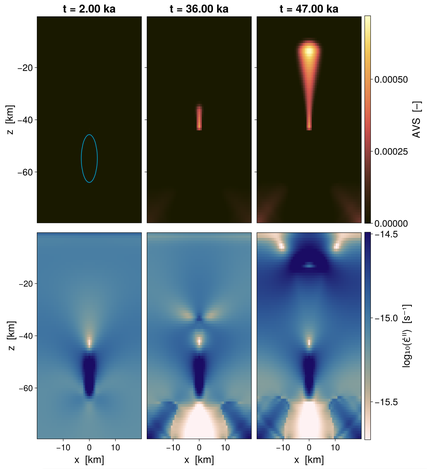
(b) Orange arrows show ∂F/∂u, projected onto R=0 as ψ. Green arrows show $K = -\psi^T B$ — the parameter update δm. White lines trace the inversion toward u_obs.

Petrological Experiment



*How can we **optimize** the diffusion coefficient to increase the **temperature** and **reduce gradients** inside the sample?*

***dF/dD** reveals where changes to the diffusion coefficients have the greatest effect:*
*- **increase** D along the **thermal pathway***
*- **decrease** it in the **sample walls** to trap heat*



Next Steps:

*combine all of the physics to get a **poro-thermo-visco-elasto-plastic** solver*

*for example: **tensile plasticity** induced fluid ascent, which parameters control the **speed** and **width**?*

References

[1] *Reuber, G. S., Popov, A. A., & Kaus, B. J. (2018). Deriving scaling laws in geodynamics using adjoint gradients. Tectonophysics, 746, 352-363.*