Hilbert transform applied to internal waves issues

Matthieu J. Mercier, Nicolas B. Garnier & Thierry Dauxois

Université de Lyon, Laboratoire de Physique de l'École Normale Supérieure de Lyon, CNRS.
46 allée d'Italie, 69364 LYON Cedex 07.
email: firstname.lastname@ens-lyon.fr

ECOLE NORMALE SUPERIEURE DE LYON

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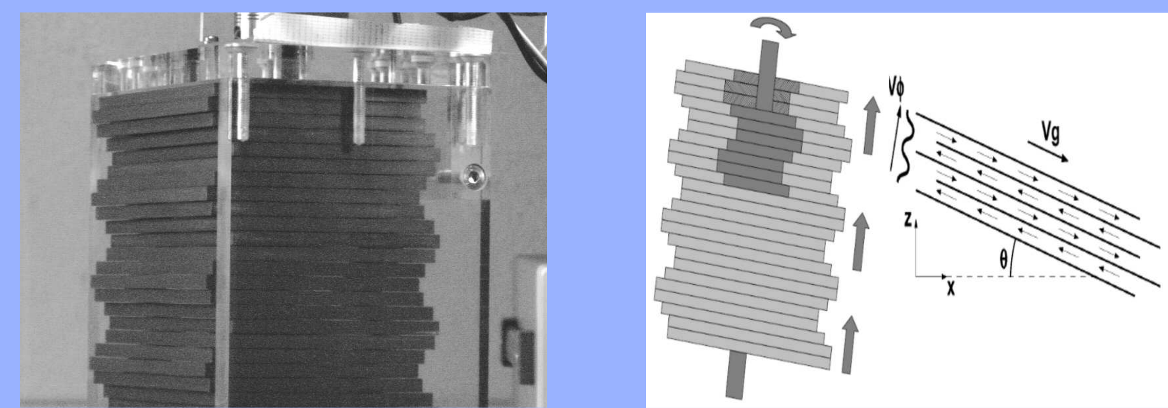
1. A new internal plane wave generator

Principle: the generator imposes the horizontal velocity at a boundary

$v_x = v_0 e^{i(\omega t - \frac{2\pi}{\lambda_c} z)} \quad \Rightarrow \quad k_z = k\cos\theta = \frac{2\pi}{\lambda_c}$

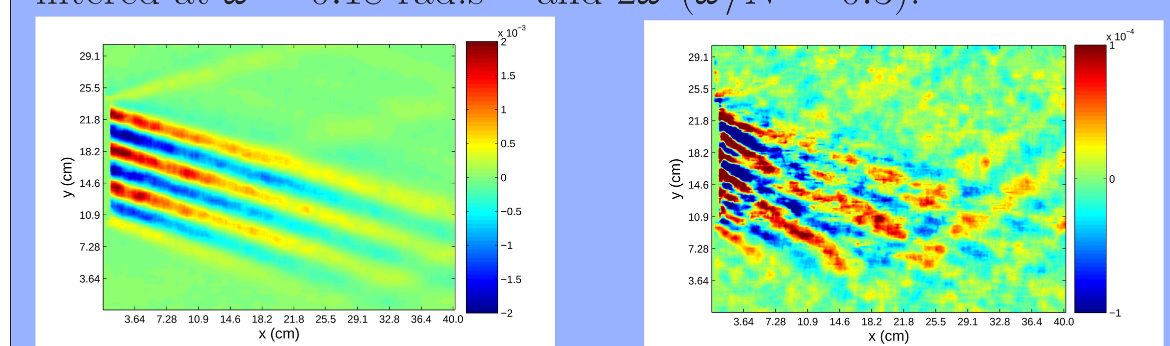
Phase constraint allows emission of only one beam (instead of four possible for a given ω).



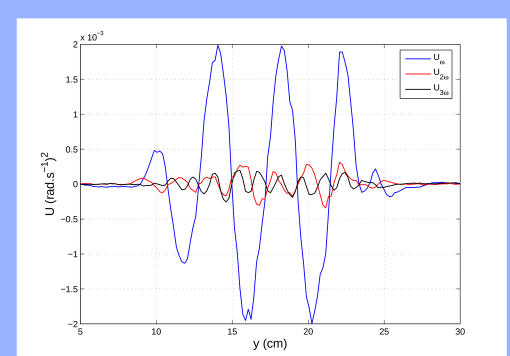
Picture and scheme of the wavemaker.

Emitted waves:

Temporal harmonicity. Horizontal gradient in ΔN^2 (rad.s^{-1})^2 filtered at $\omega = 0.18$ rad.s^{-1} and 2ω ($\omega/N = 0.3$).



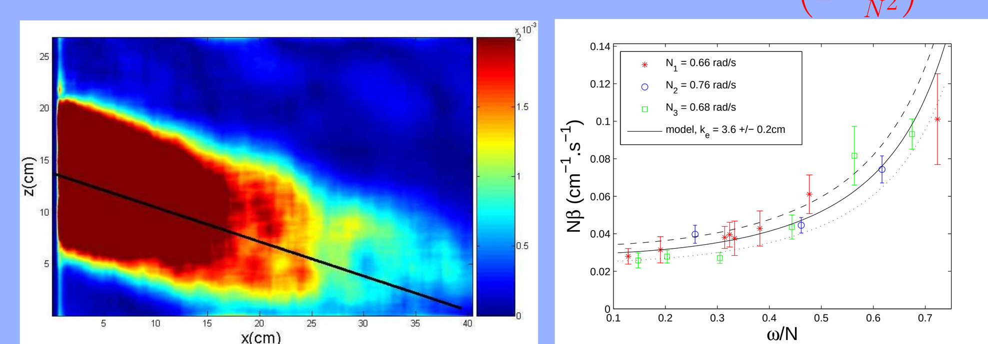
Spatial monochromaticity. Vertical profiles of emited waves filtered at ω, 2ω et 3ω (in ΔN^2).



Dissipation:

exponential decay along propagation

$$\psi \simeq e^{-\beta\xi} e^{i(\omega t - k\eta)} \text{ with } N\beta = \frac{\nu k_c^3}{2} \frac{1}{\left(1-\frac{\omega^2}{N^2}\right)^2}$$



(left) Horizontal gradient's envelope filtered at ω in ΔN^2.
(right) Test of the law $N\beta = f(\omega/N)$ for different N.

Gostiaux, Didelle, Mercier and Dauxois, Experiment in Fluids 42, 123 (2007)

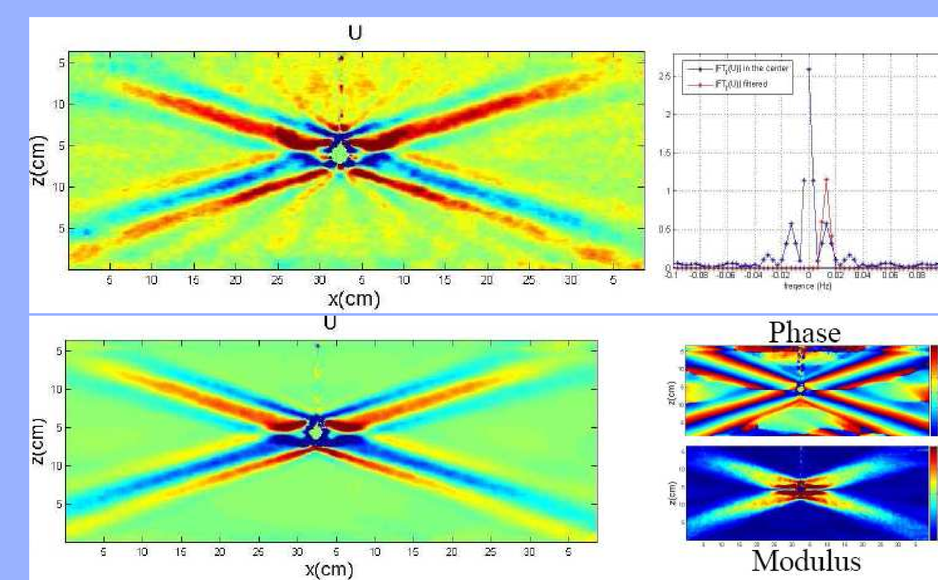
2. A new tool for analysis: Hilbert transform

$$\mathbf{U = A\cos(\omega t \pm kx) \Rightarrow \tilde{\tilde{U}} = Ae^{i(\omega t - kx)} \text{ or } Ae^{i(\omega t + kx)}}$$

Example to illustrate the technique: oscillating cylinder.

step 1: $\tilde{U} = Ae^{i(\omega t \pm kx)}$

Fourier transform (in time), filtering of negative frequencies and inverse Fourier transform.



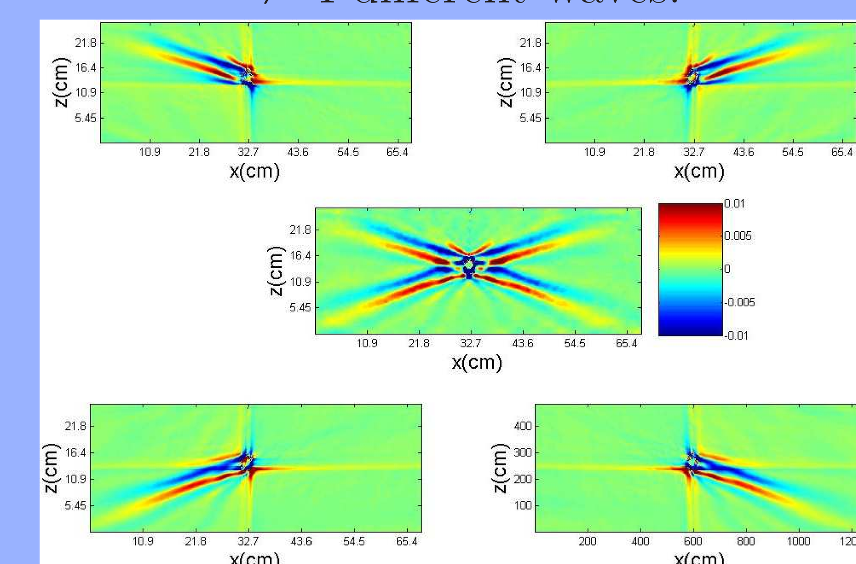
(top) Horizontal gradient and its temporal spectrum.
(down) $Re(\tilde{U})$, $|\tilde{U}|$ and $Arg(\tilde{u})$ in ΔN^2 (rad.s^{-1})^2.

- Selective filtering of harmonics
- Envelope and phase obtained easily
- How to discriminate beams?

step 2: $\tilde{\tilde{U}} = Ae^{i(\omega t - kx)}$ or $Ae^{i(\omega t + kx)}$

Fourier transform (in space), filtering of negative/positive wavenumbers, and inverse Fourier transform. With 2D-internal waves,

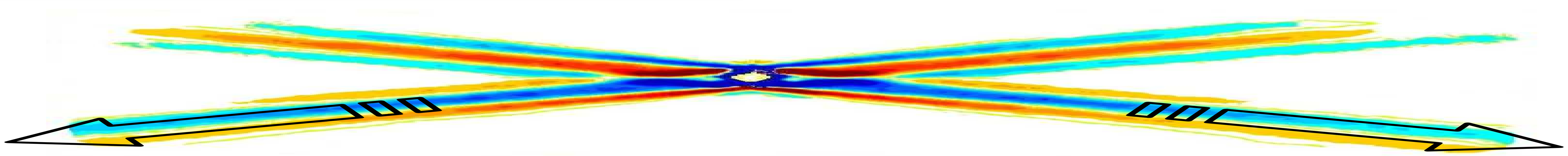
$$\tilde{U}(x,z,t) = A(x,z,t)e^{i(\omega t \pm k_x x \pm k_z z)}$$
$$\Rightarrow \text{ 4 different waves!}$$



Spatial decomposition of the horizontal gradient in ΔN^2 (rad.s^{-1})^2.

- Each wave isolated in a systematic manner

Mercier, Garnier and Dauxois, Physics of Fluids, submitted (2008)

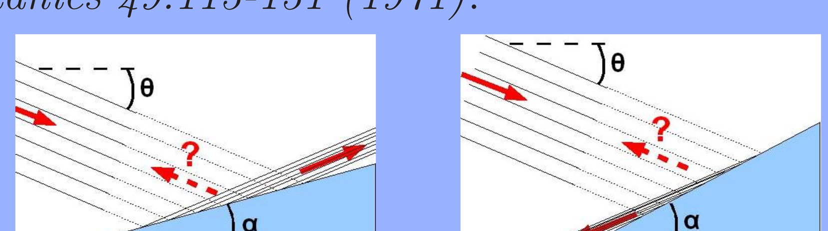


3. Reflexion, an old topic ?

Critical reflexion: Problem studied analytically and experimentally, leading to harmonics creation and turbulence. Recent works: *Dauxois and Young, Journal of Fluid Mechanics 390, 271 (1999)* or *Gostiaux et al, Physics of Fluids 18, 056602 (2006)*.

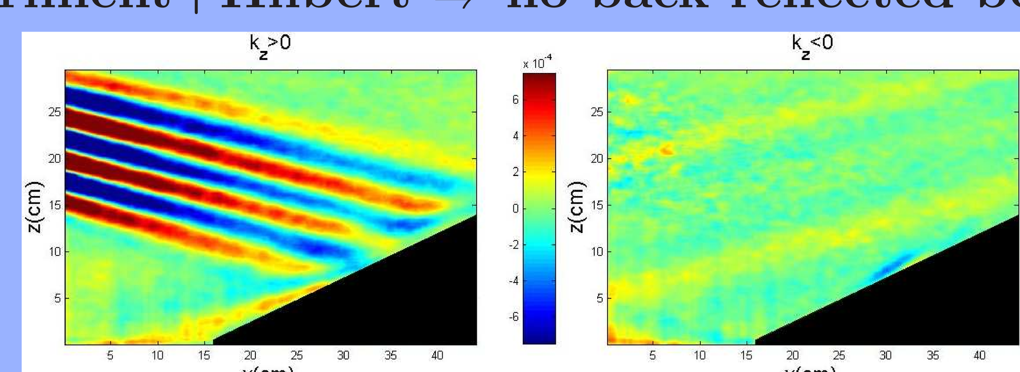
Back-reflected waves:

an old problem raised by Baines in *Journal of Fluid Mechanics 49:113-131 (1971)*.



Principle, in the case $\theta > \alpha$ (left) and $\theta < \alpha$ (right).

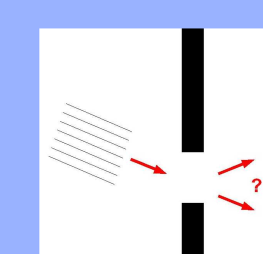
Experiment+Hilbert $\Rightarrow$ no back-reflected beam



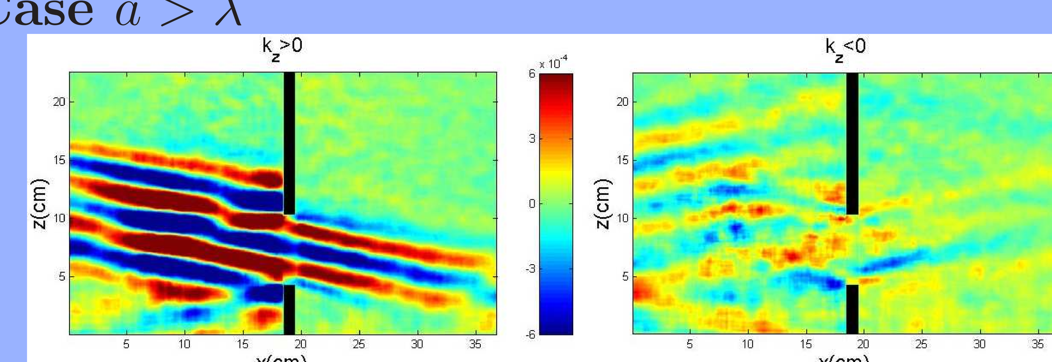
Case $\theta < \alpha$, horizontal gradient in ΔN^2. Same results for $\theta > \alpha$.

4. Diffraction, an unsual problem

What to expect after the slit of width a ? Is a/λ a key parameter ?

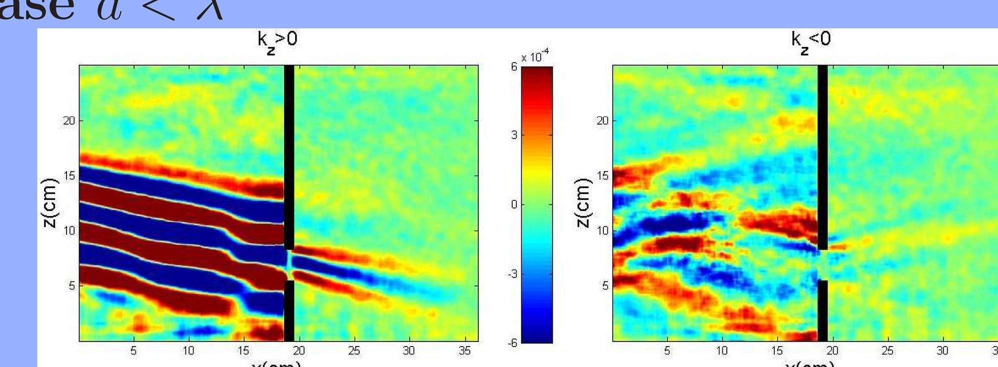


Case $a > \lambda$



Horizontal gradient of downward (left) or upward (right) propagating waves (in ΔN^2).

Case $a < \lambda$



Horizontal gradient of downward (left) or upward (right) propagating waves (in ΔN^2).

Perspectives

- **1**→ to generate different profiles (mode 1, self-similar,···)
- **2**→ to generalize the Hilbert transform to non-stationnary processes
- **3**→ to study reflection on concave/convex surfaces
- **4**→ to develop diffraction's theory