Using Kalman smoother to derive daily gravity field solutions from GRACE L1B data

Enrico Kurtenbach, Torsten Mayer-Gürr, and Annette Eicker

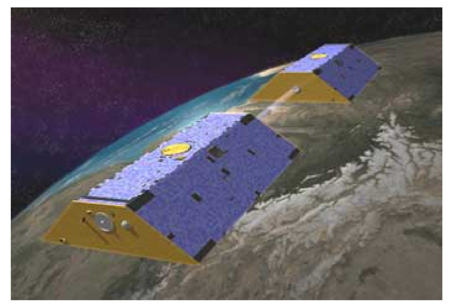
Institute of Geodesy and Geoinformation, Department of Astronomical, Physical and Mathematical Geodesy, University of Bonn, Germany

Abstract

Different GRACE data analysis centers provide temporal variations of the Earth's gravity field as monthly, 10-daily or weekly mean fields. These solutions are derived independently for each time span, i.e. no correlation between the analysed batches is considered. Following this procedure, an increase in temporal resolution is accompanied by a loss in accuracy. To avoid this problem, Kurtenbach et al. (2009) presented a new approach, which takes into account the temporal correlations of the gravity field variations thus enabling the enhancement of the temporal resolution up to daily snapshots. The GRACE Level-1B (L1B) instrument data processing is performed within the framework of a Kalman filter estimation procedure. In this contribution an improved approach is presented, which takes into account the full temporal and spatial correlation pattern of the expected gravity field signal. The required information in terms of an empirical auto-covariance function is derived in this approach from atmospheric, oceanic, and hydrological model data.

The Earth's temporal gravity field as linear dynamic system

Observation model



Analysing GRACE observations leads to a system of observation equations

$$\mathbf{A}_t\mathbf{x}_t = \mathbf{y}_t + \mathbf{v}_t,$$

with the design matrix $\mathbf{A}_t$, the vector of unknowns $\mathbf{x}_t$, the observations $\mathbf{y}_t$ and the noise vector $\mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$

Least squares adjustment leads to a system of normal equations

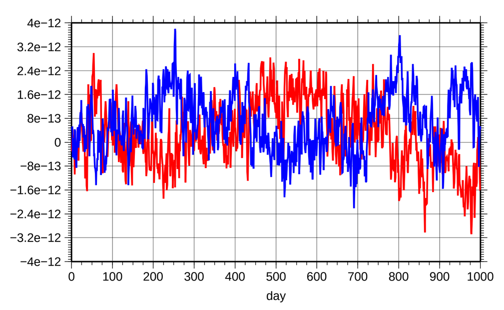
$$\mathbf{N}_t\mathbf{x}_t = \mathbf{n}_t,$$

which can be solved separately for each epoch t:

$$\hat{\mathbf{x}}_t = \mathbf{N}_t^{-1}\mathbf{n}_t.$$

Process model

Assuming that the gravity field parameters cannot change within an arbitrary range, the solution $\mathbf{x}_t$ on the current day t can be predicted from the previous at $t-1$ according to



$$\mathbf{x}_{t+1} = \mathbf{B}\mathbf{x}_t + w,$$

with the dynamic of the process $\mathbf{B}$ and the prediction noise vector $w \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$. Hereby, a stationary, first order Markov process is implicitely assumed. In the following, two different process models, (M1) and (M2), are presented and compared.

Combining observation model and process model – the Kalman filter and smoother

The common tool to combine the information from observations $\mathbf{y}_t = \mathbf{A}_t\mathbf{x}_t$, here from GRACE, and from a process model $\mathbf{x}_{t+1} = \mathbf{B}\mathbf{x}_t$ is the Kalman filter, which provides an optimal weighting of both in a least-squares sense.
Because of the postprocessing analysis of the GRACE observations, not only a filter but also a fixed-interval smoother, here the computational efficient RTS-smoother (Rauch et al., 1965), is applied to use all observations in the given interval $t \in [0, \ldots, T]$ when estimating the state at epoch t.

Prediction

$$\hat{\mathbf{x}}_{t+1}^- = \mathbf{B}\hat{\mathbf{x}}_t^+$$

$$\mathbf{\Sigma}_{t+1}^- = \mathbf{B}\mathbf{\Sigma}_t^+\mathbf{B}^T + \mathbf{Q}$$

Update

$$\mathbf{\Sigma}_t^+ = \left(\left(\mathbf{\Sigma}_t^-\right)^{-1} + \mathbf{N}_t\right)^{-1}$$

$$\hat{\mathbf{x}}_t^+ = \hat{\mathbf{x}}_t^- + \mathbf{\Sigma}_t^+\left(\mathbf{n}_t - \mathbf{N}_t\hat{\mathbf{x}}_t^-\right)$$

Initial values for $\hat{\mathbf{x}}_0^+$ and $\mathbf{\Sigma}_0^+$

M1 – Random walk

Kurtenbach et al. (2009) used the simple assumption for the process dynamic

$$\mathbf{B} = \mathbf{I},$$

where the prediction error can be modeled as covariance matrix of the first differences of the considered dynamic process:

$$\mathbf{Q} = \mathcal{C}\left\{(\mathbf{x}_{t+1} - \mathbf{x}_t)\right\}.$$

Instead of the isotropic covariance function used by Kurtenbach et al. (2009), in this study a full covariance matrix $\mathbf{Q}$ of the prediction error is derived from geophysical models.

M2 – Least squares prediction

The full correlation pattern between two states of a stationary, first order Markov process can be described by

$$\mathcal{C}\left\{\begin{pmatrix}\mathbf{x}_{t+1}\\ \mathbf{x}_t\end{pmatrix}\right\} = \begin{pmatrix}\mathbf{\Sigma} & \mathbf{\Sigma}_\Delta^T\\ \mathbf{\Sigma}_\Delta & \mathbf{\Sigma}\end{pmatrix}$$

According to Moritz (1980) a linear least-squares predictor $\hat{\mathbf{x}}_{t+1} = \mathbf{B}\mathbf{x}_t$ can be found as

$$\mathbf{B} = \mathbf{\Sigma}_\Delta\mathbf{\Sigma}^{-1}$$

with the covariance matrix of the prediction noise $\mathbf{w}$:

$$\mathbf{Q} = \mathbf{\Sigma} - \mathbf{\Sigma}_\Delta\mathbf{\Sigma}^{-1}\mathbf{\Sigma}_\Delta^T.$$

Simulation study

Process dynamic

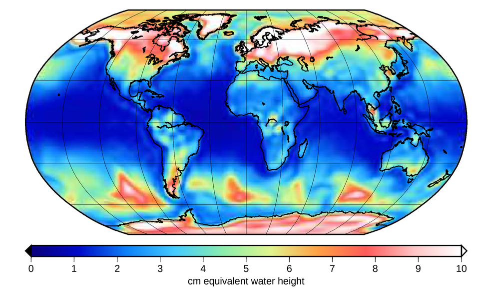
Process dynamic $\mathbf{B}$ and process noise $\mathbf{Q}$

- for M1 (random walk): empirical auto-covariance $\mathcal{C}\{x_{t+1} - x_t, x_{t+1} - x_t\}$
- for M2 (least squares prediction): empirical auto-covariance $\mathbf{\Sigma} = \mathcal{C}\{\mathbf{x}_t, \mathbf{x}_t\}$ and cross-covariance $\mathbf{\Sigma}_\Delta = \mathcal{C}\{\mathbf{x}_{t+1}, \mathbf{x}_t\}$

from AOH of 01/1976 to 12/2006.

Geophysical models

Daily mass variations of
Atmosphere from AOD1B
Ocean from AOD1B
Hydrology from WGHM



RMS of temporal variability of AOH

GRACE observations

Simulate GRACE observations

- orbits,
- K-band ranges, and
- accelerometer

for 01/2007 to 12/2007.

Build normal equations

$$\mathbf{N}_t\mathbf{x}_t = \mathbf{n}_t$$

(degree/order 40) for each day.

Run process $\mathbf{x}_{t+1} = \mathbf{B}\mathbf{x}_t + \mathbf{w}$ with random prediction noise $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$ for $T = 1000$ days

Kalman filter/smoother
$\hat{\mathbf{x}}_0^+ = \text{AOH}(t_0)$
$\mathbf{P}_0^+ = \mathbf{\Sigma}$

Simulate dynamic process

Figure: Progress of exemplary coefficients ($c_{7,1}$, $s_{13,3}$, and $c_{35,10}$)

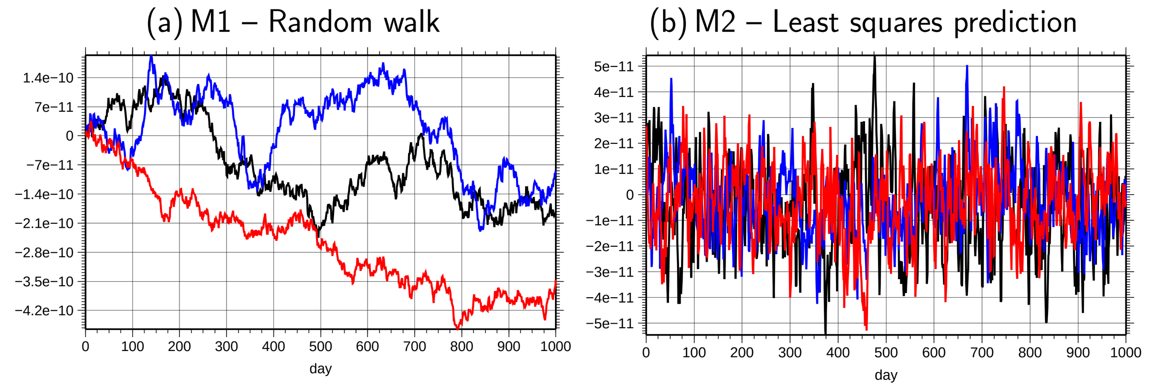


Figure: Evaluate states $\mathbf{x}_t$ at point $\mathbf{r} = (-60, 0)$ in the amazon basin

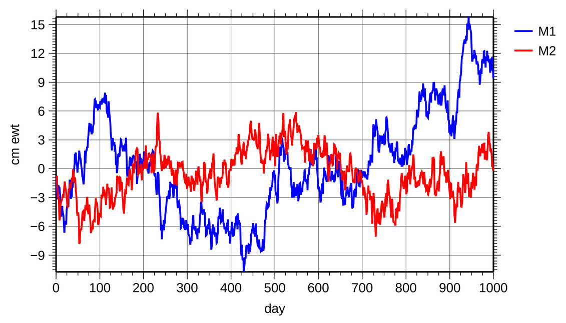


Figure: Evaluate states $\mathbf{x}_t$ at point $\mathbf{r} = (-0, -50)$ in the south atlantic

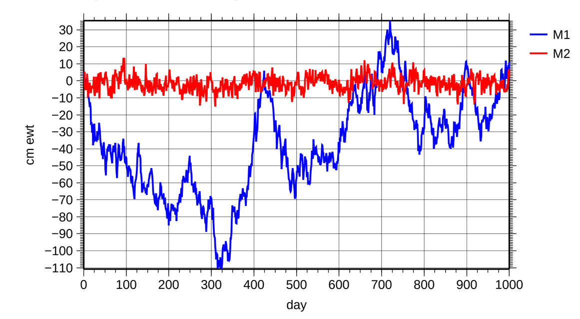
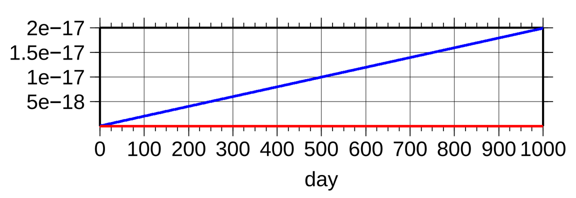


Figure: Trace of $\mathcal{C}\{\mathbf{x}_t\}$ for M1 and M2



The prediction accuracy $\mathrm{tr}[\mathcal{C}\{\mathbf{x}_t\}]$ of M1 exceeds all bounds, whereas M2 represents a steady state process when $\mathbf{P}_0^+ = \mathbf{\Sigma}$.

Analysis of simulated GRACE observations

Figure: Evaluate states $\mathbf{x}_t$ at amazon (left) and south atlantic (right)

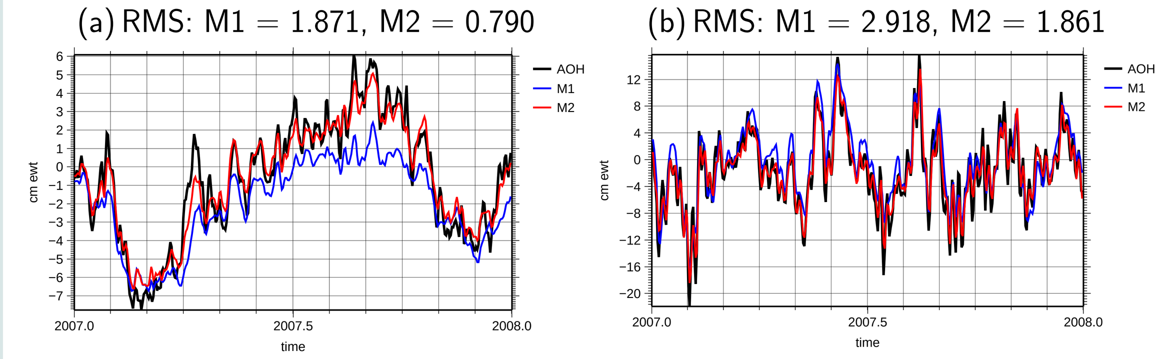
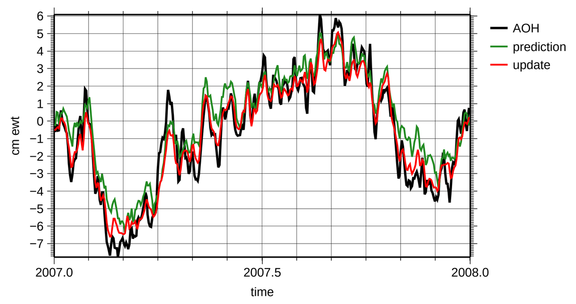
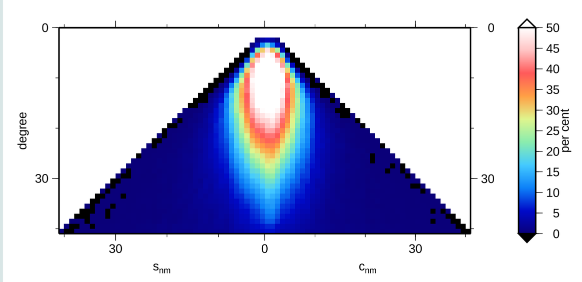


Figure: Predicted state $\hat{\mathbf{x}}_t^-$ and updated state $\hat{\mathbf{x}}_t^+$ for M2



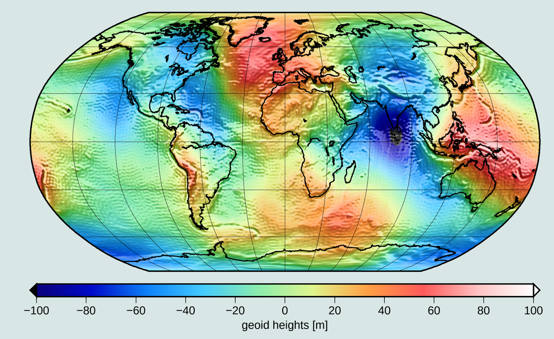
The loss of information by model simplification from AOH to $\mathbf{B}$ is small. The chosen process noise $\mathbf{Q}$ gives good results as the addition of GRACE observations leads to a better fitting to the reference signal.

Figure: Exemplary contribution of observations to estimated state



The Kalman filter weights observations and underlying process model. The contribution of the observations to the daily estimate in per cent is shown for each potential coefficient for an arbitrary day.

IGG's new GRACE gravity field release ITG-Grace2010



The presented approach M2 was used to derive daily solutions of degree/order 40 in the current GRACE gravity field release ITG-Grace-2010 of the IGG at University of Bonn. These daily solutions were also used as an improved dealiasing product when deriving the monthly solutions. For further information see

`http://www.igg.uni-bonn.de/apmg/index.php?id=itg-grace2010`

Institute of Geodesy and Geoinformation
Department of Astronomical, Physical and Mathematical Geodesy
University of Bonn

Contact:
Enrico Kurtenbach
kurtenbach@igg.uni-bonn.de

Nussallee 17
D-53115 Bonn
www.igg.uni-bonn.de/apmg/

universität bonn