Stability of Submesoscale Wake Vortices in a Rotating and Stratified Shallow-Water Layer: Laboratory Experiments

Ayah Lazar [1,2], Alexandre Stegner [2], Rui Caldeira [3], Samuel Viboud [4]
and Henri Didelle [4]

contact: AYAHLAZAR@gmail.com
Affiliations:
1) LMD, Ecole Polytechnique, LMD, IPSL, Palaiseau, France
2) Geophysics and Planetary Sciences,Tel-Aviv University, Tel-Aviv, Israel
3) Center of Marine and Environmental Research (CIIMAR) Portugal
4) LEGI-Coriolis, CNRS, Grenoble, France

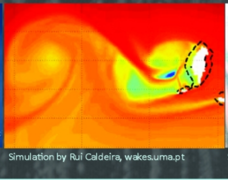
1 Abstract

Laboratory experiments were performed at the Coriolis rotating platform to study the three-dimensional instabilities of intense wake vortices. A linear salt stratification was set in the upper layer on top of a thick barotropic layer, to mimic the oceanic thermocline on top of the deep-ocean. In order to reproduce the dynamic of a surface current interacting with an isolated and steep island (or archipelago), a few obstacles, immersed only in the top stratified layer, were towed in various velocities and stratifications. Generating a wake of submesoscale eddies (i.e. radius smaller than the local deformation radius).

This experiment is the first laboratory study, at large Reynolds numbers (Re=3000-50000), exploring the impact of the thermocline stratification on the stability of intense shallow-water anticyclones. It is well known that inertial and/or elliptical instabilities may strongly destabilize intense anticyclonic eddies (with vorticities of $\zeta/f < -1$, where ζ is the relative vorticity and f is the Coriolis parameter). However, we found that several anticyclones remain stable even for intense negative vorticity values, even up to $\zeta/f \approx -3.5$ in strong stratification conditions (i.e. large burger number).

Linear stability analysis shows that the growth of three-dimensional perturbations, is mainly controlled by: the Burger number, the Rossby number, and the Ekman number. Both laboratory and theoretical studies confirm a large stability region for the submesoscale anticyclonic vortices, particularly when the Rossby and the Burger numbers are large.

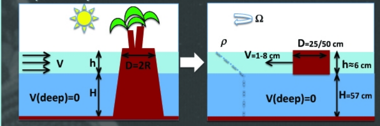

Photograph from NASA


Simulation by Rui Caldeira, wakes.uma.pt


S. Teinturier et al, 2008

2 Experimental set-up

We model the surface intensified flow on an island by dragging a cylinder in the stratified shallow upper layer of water in the 14m diameter LEGI-Coriolis rotating tank. The upper layer is seeded with particles which are illuminated and captured by 4 different cameras



$14\ [m]$ diameter tank


From LEGI-Coriolis website

particles for PIV processing

Towing the cylinder in the tank


Taken at the experiment

Shallow water configuration: $\delta = \frac{h}{R} = 0.1 - 0.5 < 1$

Different towing speeds produce vortices of different intensities:

$$\frac{V}{Rf} = 0.1 - 10 \longrightarrow |Ro| = \frac{|Vmax|}{R_{max}f} = 0.1 - 4$$

Known Ekman as a function of height: $$\boxed{1/Ek = \frac{fh^2}{\nu} = 300 - 600}$$

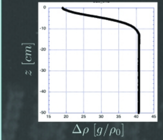
Impact of the stratification and vertical confinement ?

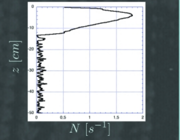
$$\boxed{N/f = 7 - 20 \qquad \sqrt{Bu} = N\delta = \frac{R_d}{R} = 2 - 10}$$

Careful to stay in the sub-critical regime: $F_D = \frac{Ro}{\sqrt{Bu}} < 1$

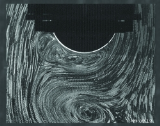
3 Data

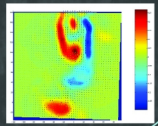
Measured density profile of example exp. C25_012 → Calculated stratification





Particle displacement (ten consecutive frames) → Velocity and vorticity fields obtained by PIV software





4 Linear stability analysis

Assumptions:

- Mean flow and perturbations are axisymmetric $\partial_\theta \to 0$
- Boussinesq approximation
- Prandtl number ~ 1
- Small vertical scale compared to horizontal $\nabla^2 \to \partial_z^2$

Linearized equations:

$$\begin{aligned} \partial_t u - \frac{2v\bar{V}}{r} - fv &= -\frac{1}{\rho_0}\partial_r p + \nu\partial_z^2 u \\ \partial_t v + \left(\bar{\zeta} + f\right)u &= \nu\partial_z^2 v \\ \partial_t w - b &= -\frac{1}{\rho_0}\partial_z p + \nu\partial_z^2 w \\ \partial_t b + N^2 w &= \nu\partial_z^2 b \\ \frac{1}{r}\partial_r(ru) + \partial_z w &= 0. \end{aligned}$$

Defining a stream function and looking for a normal mode solution:

$$u = \partial_z\psi \qquad \psi \sim \phi(r)e^{i(mz-wt)}$$
$$w = -\tfrac{1}{r}\partial_r(r\psi) \qquad \omega = \omega_{real} + i\sigma$$

The vertical wave number is quantized due to vertical confinement:

$$w(0) = w(h) = 0 \to m = n\pi/h$$

We get to an Eigenvalue problem

$$\boxed{\left[\frac{(n\pi)^2}{B_u}\left(\partial_r^2 + \frac{1}{r}\partial_r - \frac{1}{r^2}\right) - \chi(r)\right]\phi = -\tilde{\omega}^2\phi \equiv \lambda\phi}$$

For strong stratification (slightly different for non-stratified), where:

$$\tilde{\omega} = \omega_{real} + i\left(\sigma + (n\pi)^2 E_k\right)$$

$$\boxed{E_k = \frac{\nu}{fh^2}}$$

$$\boxed{Bu = \left(\frac{Nh}{fR}\right)^2}$$

$$\boxed{\chi(r) \equiv \left(\bar{\zeta} + f\right)\left(2\bar{V}/r + f\right)}$$

5 Governing parameters

The flow is characterized by:

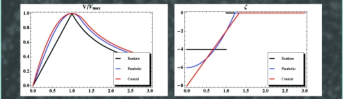
(1) The dissipation $\boxed{E_k = \frac{\nu}{fh^2}}$

(2) The ration between the radius of deformation and the vortex size $\boxed{Bu = \left(\frac{Nh}{fR}\right)^2}$

(3) And the vorticity
Vortex Rossby number or core vorticity

$$\boxed{Ro = \frac{V_{max}}{R_{max}f} \text{ or } \frac{\bar{\zeta}}{f}}$$

Q: Which one determines Stability?

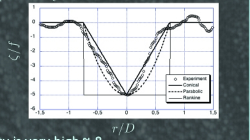


A: Stability are the same for different types of vortices when using the vortex Rossby number, and very different when the core vorticity is used

$$\Downarrow$$

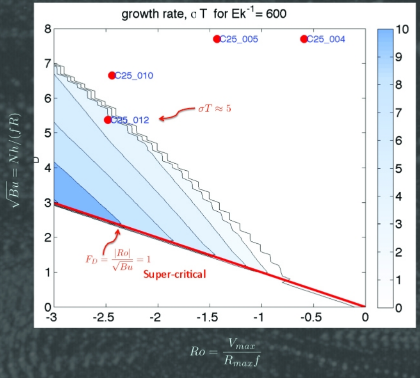
$$\boxed{Ro = \frac{V_{max}}{R_{max}f}}$$

Anticyclone vorticity profile (exp C25_012, for example) is parabolic/conical:



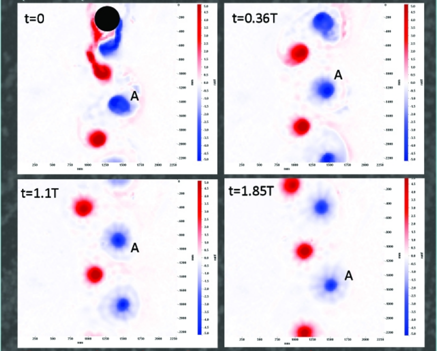
Core vorticity is very high ~-8
Vortex Rossby number is ~-2.5

6 Stability Diagram



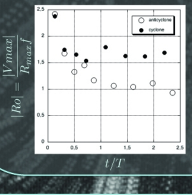
7 Evolution of an unstable wake (C25_012)

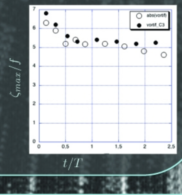
Vorticity field with time (T is the rotation period), an anticyclone (marked A) is followed



The anticyclones decay, from the periphery inwards, while the cyclones persist. Seen better in a time series:

However, the core vorticity decays only slightly for cyclones and anticyclones alike:





8 Conclusions and Achievements

- Our simple model predicts stability well.
- The parameters that control the instability are the vortex Rossby number, the Burger and the Ekman numbers.
- Anticyclones with strong core vorticities may still be stable, because their respective vortex Rossby number is low enough (this ratio depends strongly on the vortex shape).
- The stratification moderates the instability
- The larger the radius of a vortex (compared to the deformation radius) the more unstable it is
- A signature of inertial Instability – the Vortex Rossby number of the unstable anticyclone decays while the core vorticity does not.