The Dependence of the Atlantic Overturning Circulation on Meridional Density Gradients

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1. Introduction

Due to the complexity of the equations governing the ocean circulation, great effort has been invested in seeking a **simple relation** between the Atlantic **meridional overturning circulation** (**MOC**) and basic metrics of the ocean state. Particular attention has been given to the MOC dependence on the **meridional density gradient** (**MDG**). The classical view of the MOC as gravity current depending linearly on the MDG has been challenged in many numerical modelling studies.

Does a fixed MOC-MDG proportionality exist?

We derive a **new analytical expression** for the MOC-MDG dependence. By defining a buoyancy **"force function"** we obtain **information on both strength and structural changes of the MOC in response to MDG changes.**

2. Classical Scaling Arguments

Linear MOC-MDG relation

Scale the thermal wind equation assuming the depth scale, H, is constant and the horizontal velocity components are similar *(Robinson and Stommel 1959, Bryan and Cox 1969)*:

$$\psi_{MOC} = VLH = \frac{g\Delta_y \rho H^2}{\rho_0 f_0}$$

$V =$ meridional velocity scale
$L =$ horizontal length scale
$H =$ vertical depth scale
$g =$ gravity
$\Delta_y\rho =$ discrete MDG
$\rho_0 =$ ref. density
$f_0 =$ ref. Coriolis parameter

MOC-MDG power law relation

Extend the scaling to permit variations in H e.g under advective-diffusive balance *(Bryan and Cox 1967)*:

$$\psi_{MOC} = \left(\frac{gL^4k_v^2}{\rho_0 f_0}\right)^{1/3} \Delta_y\rho^{1/3}$$

$k_v =$ vertical diffusivity

- Substantial support for both the linear MOC-MDG relation *(Hughes and Weaver 1994, Rhamstorf 1996, Thorpe et al. 2001, Griesel and Maqueda 2006, Dijkstra 2008)* and a power law relation *(Winton 1996, Marotzke 1997, Park and Bryan 2000)* from numerical models.
- Numerical evidence also suggests the MOC may even be anti-correlated with certain measures of the MDG *(Nilsson et al. 2003, de Boer et al. 2010)*.

Uncertainty in:
1. Validity of $U \sim V$ assumption
2. Relevant H (Thermocline depth? Depth of max. MOC?)
3. Relevant $\Delta_y\rho$ (Surface? Depth averaged?)

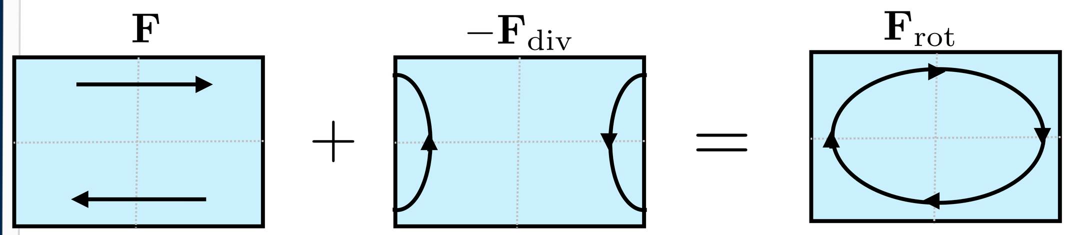
3. Dynamically Important Buoyancy Forcing?

Any force, $\mathbf{F}$, can be decomposed into rotational and divergent parts:

$$\left.\frac{\partial \mathbf{u}}{\partial t}\right|_F + \frac{\nabla p_F}{\rho_0} = \frac{\mathbf{F}}{\rho_0} \quad \text{where} \quad \mathbf{F} = \mathbf{F}_{rot} + \mathbf{F}_{div} \qquad (\nabla\cdot\mathbf{F}_{rot} = 0) \quad (\nabla\cdot\mathbf{u} = 0)$$

projects entirely onto the local acceleration

projects entirely onto the pressure gradient



We isolate the purely rotational part of the buoyancy forcing which projects directly onto the Eulerian acceleration.

FIG 1: 2D example: $\mathbf{F}_{div}$ removes/returns fluid from/to the boundary satisfying continuity and the kinematic boundary condition. $\mathbf{F}_{rot}$ is wholly responsible for the net acceleration of the fluid.

4. Mapping the Rotational Buoyancy Forcing onto the MOC

Introduce a "force function" to describe $\mathbf{F}_{rot}$ *(Marshall and Pillar 2011)*

$$\mathbf{F}_{rot} = \rho_0\nabla\times\mathbf{A}_F \qquad (\nabla\cdot\mathbf{F}_{rot} = 0)$$

$$\mathbf{A}_F = \mathbf{A}_{OT} + A_{BT}^{(z)}\frac{z}{D+\eta}\nabla_h(D+\eta) + A_{BT}^{(z)}\mathbf{k}$$

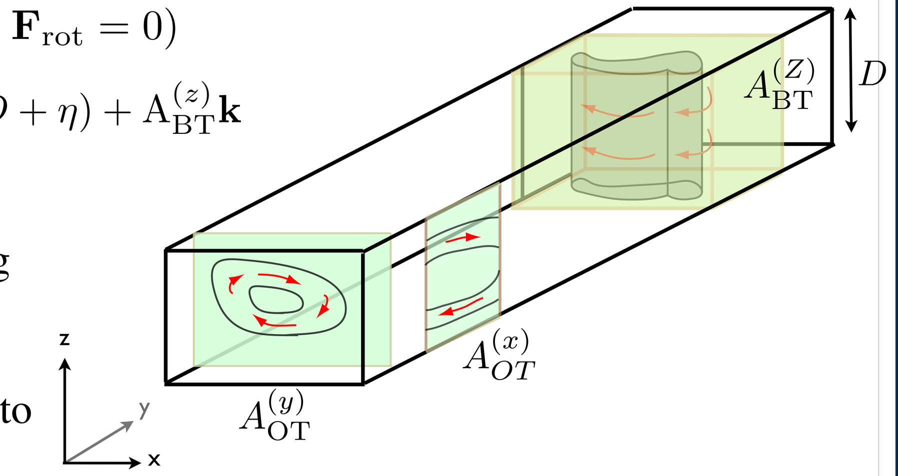


FIG 2: In the hydrostatic limit, $\mathbf{A}_F$, can be written in terms of overturning and barotropic parts. The zonal overturning force function, $A_{OT}^{(x)}$, describes the projection of a force onto the MOC.

We determine $A_{OT}^{(x)}$ associated with the buoyancy force.

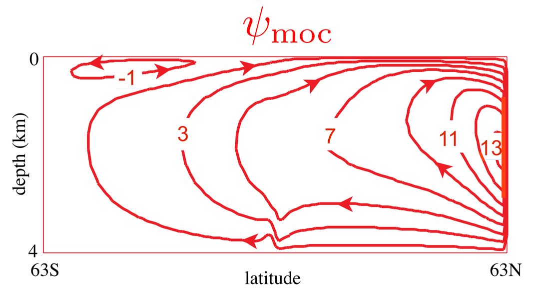
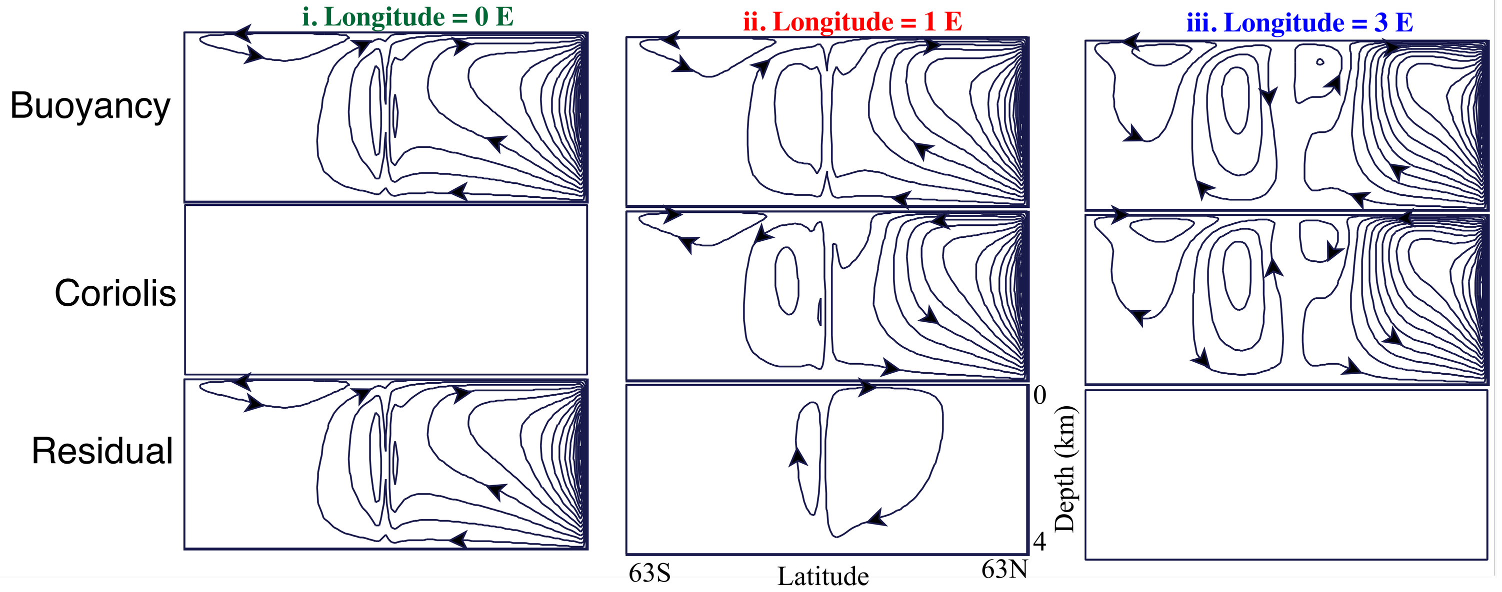


FIG 3: Strong similarity has been shown between the MOC and buoyancy $A_{OT}^{(x)}$ in an idealized numerical model of the MOC *(Marshall and Pillar 2011)*. The rotational buoyancy force accelerates the MOC at the boundaries where compensation from Coriolis is absent.



5. A New Analytical MOC-MDG Relationship

Following *Marshall and Pillar 2011:*

$$\frac{\partial^2 A_{OT}^{(x)}}{\partial z^2} = -\frac{1}{\rho_0}(\nabla\times\mathbf{F})\cdot\hat{\imath} \qquad (1)$$

$$\mathbf{F}\Big|_{buoyancy} = -g\rho'\mathbf{k}$$

$$A_{OT}^{(x)}\Big|_{buoyancy} = f\int_{-D}^{z}(u_g - c)\,dz \qquad \begin{array}{l} c = \text{constant} \\ u_g = \text{zonal geostrophic velocity}\end{array} \qquad (2)$$

$A_{OT}^{(x)}$ vanishes on the top and bottom boundaries:

$$A_{OT}^{(x)}\Big|_{buoyancy} = f\int_{-D}^{z}(u_g - \overline{u_g}^z)\,dz$$

Invoking integration by parts, the geostrophic and thermal wind equations we find:

$$\left.\frac{\partial\psi_{MOC}}{\partial t}\right|_{buoyancy} = \int_{x_w}^{x_e} A_{OT}^{(x)}\Big|_{buoyancy} dx = \frac{g}{\rho_0}\int_{x_w}^{x_e}\int_{-Z_{max}}^{\eta}\frac{\partial\rho}{\partial y}\,z\,dz\,dx \qquad (3)$$

(Pillar et al. 2012)

1. MOC is linearly related to the depth integrated MDG
2. Relevant $H = Z_{max}$ = depth of maximum MOC
3. Relevant $\Delta_y\rho$ = MDG integrated between the surface and the depth of the max. MOC

Consistent with *de Boer et al. 2010* but note:
$A_{OT}^{(x)}$ also gives information on **MOC structure.**

6. Simple View of the MOC

$$A_{OT}^{(x)}\Big|_{Coriolis} = -f\int_{-D}^{z}(u - \overline{u}^z)\,dz$$

$$A_{OT}^{(x)}\Big|_{Coriolis} + A_{OT}^{(x)}\Big|_{buoyancy} = -f\int_{-D}^{z}(u_{ag} - \overline{u_{ag}}^z)\,dz$$

similar to cross-frontal secondary circulations

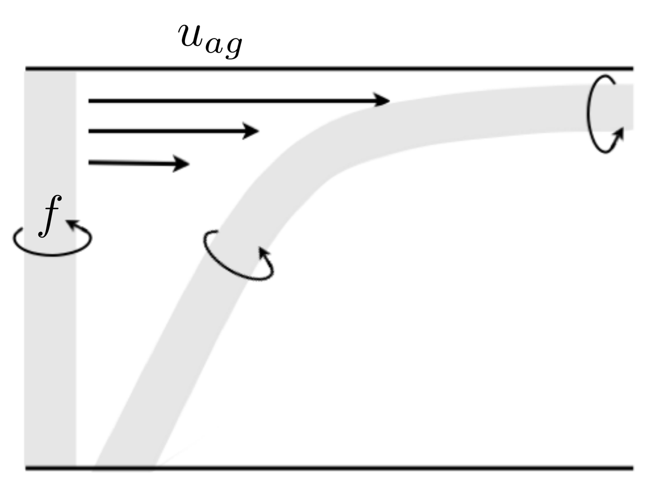


FIG 4: Rotational Coriolis and buoyancy forces are equal and opposite in the geostrophic interior. Within the viscous boundary layers thermal wind balance breaks down and the MOC is driven by shear in the ageostrophic zonal velocity tilting planetary vorticity into the y-z plane.

7. Future Work

(i) Currently seeking **empirical support** for equation (3)
- idealized numerical experiments based on *de Boer et al. 2010*
- two ensembles designed to explore the MOC over a range of Z_{max} and MDG.

Wind stress (WS) suite

$\tau \rightarrow \alpha\tau$

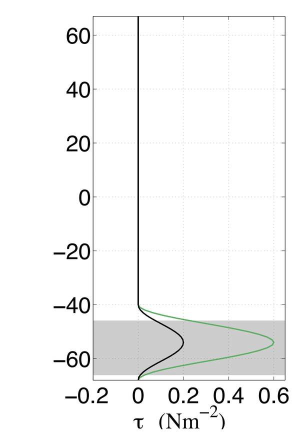


FIG 5a: wind profile for the control run (black) and the extreme high wind case ($\alpha = 3$). Stronger winds produce a deeper overturning.

Equation of state (EOS) suite

$\rho(S,\theta,p) \rightarrow \rho(S,\theta+\delta\theta,p)$

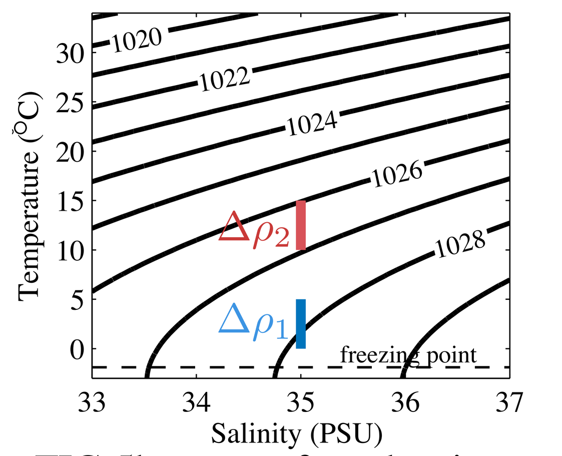


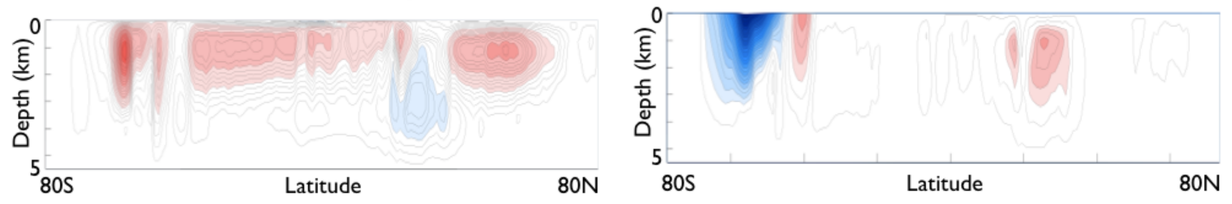
FIG 5b: sea surface density *(Jackett and McDougall 1995)*. A fixed meridional temperature gradient ($\Delta\theta = -5°C$) produces a greater MDG in a warmer ocean: $\Delta\rho_2/\Delta\rho_1 = 2.3$

(ii) **Extension to complex geometries**
- for heterogeneous bathymetry the barotropic forcing projects onto the overturning non-uniformly.
- we may distinguish between the shear and "external mode" parts:

MOC

Thermal wind component

External mode



$$\frac{\partial}{\partial t}\mathbf{\Psi}_{OT} = \mathbf{A}_{OT}$$

driven by: buoyancy forcing
Ekman forcing

$$\frac{\partial}{\partial t}\Psi_{BT}^{(z)} = A_{BT}^{(z)}$$

driven by: wind stress curl

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