

# **Computation of balanced and unbalanced** kinetic energy spectra in a limited area model



### 1 Motivation

The kinetic energy spectra based on the aircraft observations [1] describe the upper troposphere and the lower stratosphere. Most of studies of the kinetic energy distribution in NWP and climate models have therefore been focused on the energy distribution as a function of horizontal scale in the same region. The analysis has usually been based on global models which describe energy on scales greater than hundred km.

Consequently, little is known about the vertical distribution of kinetic energy below 250 hPa and at scales around and below 100 km. Furthermore, studies have usually dealt with total kinetic energy. At the same time, understanding model dynamics is easier in terms of rotational and divergent energy which correspond to balanced and unbalanced dynamics, respectively.

It is thus natural to ask the following questions:

- what part of kinetic energy is associated with balanced (vorticity) and what with unbalanced motions (divergence) and
- how does the energy partitioning between divergent and rotational energy change at different altitudes as the horizontal scale becomes smaller?

#### 2 Model, dataset and methodology

- Spectral limited area model ALADIN/SI (CY35T1)
- 4.4 km horizontal resolution, 43 vertical levels
- 450 grid points in x and 432 in y direction
- Domain size: L<sub>x</sub> = 1930 km, L<sub>y</sub> = 1850 km
- Initial and boundary conditions from ECMWF
- July 2007, 2 runs per day, 6-hour forecasts
- Fields are periodized in extension zone (11 points)
- Vorticity ( $\zeta$ ) and divergence ( $\delta$ ) are computed from wind components in spectral space:

$$\hat{\delta_{kl}^z} = ik\hat{u}_{kl}^z + il\hat{v}_{kl}^z$$

$$\hat{\hat{v}}_{kl}^z = ik\hat{v}_{kl}^z - il\hat{u}_{kl}^z$$

• The divergent and the rotational part of kinetic energy are defined as:

$$E_{DIV,kl}^{z} = \frac{1}{2} \frac{\hat{\delta_{kl}^{z}} \hat{\delta_{kl}^{z}}^{*}}{k^{2} + l^{2}}$$
$$E_{VOR,kl}^{z} = \frac{1}{2} \frac{\hat{\zeta_{kl}^{z}} \hat{\zeta_{kl}^{z}}^{*}}{k^{2} + l^{2}}$$

• To test the sensitivity of the results to the periodization method, these results including the extension zone were compared to the results obtained by detrending [2] the limited area fields, a method commonly used for the computation of LAM spectra. In the notation used, k and l are zonal and meridional wavenumbers. K is total horizontal wavenumber,  $K^2 = k^2 + l^2$ , and  $\lambda$  is horizontal wavelength,  $\lambda = 2\pi/K$ .)



*Figure 1*: *Model domain and orography.* The extension zone is denoted with the black line.



Figure 2: KE spectrum, obtained as a sum of rotational and divergent energy and KE spectrum, obtained from u and v wind components, for a random case.

[2] Errico, R. M. 1985. Spectra computed from a limited area grid. *Mon. Wea. Rev.* **113**, 1554-1562. [3] Lindborg, E. 1999. Can the atmospheric kinetic energy spectrum be explained by two-dimensional turbulence?. J. Fluid Mech. **388**, 259–288.

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#### 3 Quantification of unbalanced energy



*Figure 3*: Distribution of divergent energy contribution with respect to altitude and horizontal scale. Divergent energy is expressed as the fraction of the total kinetic energy at each model level.

There is a significant horizontal and vertical variability of the unbalanced (divergent) energy percentage. It varies from 80% in the PBL below 20 km scale and in the stratosphere below 100 km to 10% and less above 500 km scales.

Figure 4 is a summary of Figure 3, the averaging is done over all scales below 300 km. The results are shown for both periodization methods.

The main properties of the results are robust to the applied method for the periodization of the limited area fields. In the upper troposphere and in the stratosphere the relative percentage of the divergent energy is slightly smaller with the extension zone for reasons not completely clear and most likely related to the properties of both methods.





*Figure 4*: Vertical distribution of the average fraction of divergent energy in the total kinetic energy for both periodization methods.





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#### 4 Kinetic energy spectra in ALADIN

*Figure 6*: Energy spectra averaged over model levels between 9 and 11 km. VOR denotes rotational kinetic energy, DIV divergent kinetic energy, VOR+DIV denotes total kinetic energy as a sum of the rotational and divergent components whereas U+V stands for the total kinetic energy computed from the velocity components. Short dotted lines have slopes K<sup>-3</sup> and K<sup>-5/3</sup>. The dashed line is a functional fit [3] to aircraft observations.

In the mesoscale region of our interest, the total EK spectrum is very well described by the analytical fit to the data derived by Lindborg (1999). The departure from the Lindborg model occurs between 30 and 20 km scale. We conclude that the effective resolution of the applied ALADIN/SI model is about 6-7 $\Delta x$ .



*Figure 7*: Monthly and vertically averaged spectra of rotational, divergent and total kinetic energy in (a) the free troposphere, (b) the planetary boundary layer (PBL) and (c) the stratosphere.

The divergent energy spectra are in all layers characterised by shallower slopes than the rotational energy spectra.

Through most of the troposphere, the total kinetic energy spectrum at scales around 100 km and smaller is reasonably well characterised by the K<sup>-5/3</sup> power-law while in the boundary layer, the spectrum becomes shallower.

#### 5 Conclusions

- upper troposphere towards 100 hPa.
- In two layers, at approximately 900 hPa and between 500 hPa and 400 hPa, resolution.
- the rotational energy spectra.



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• At scales below 300 km, about 50% of kinetic energy in the free troposphere is divergent energy. The percentage increases towards 70% at the surface and in the

rotational kinetic energy dominates on all scales larger than the model's effective

• The maximal percentage of divergent energy in the total kinetic energy at the same level is found at stratospheric levels around 100 hPa and at scales below 100 km, which are not represented by the global models analysed so far. This result calls for further studies with high-resolution models and comparisons with observations. • At all levels, the divergent energy spectra are characterised by shallower slopes than



**References:** 

<sup>[1]</sup> Nastrom, G. D. and Gage, K. S. 1985. A climatology of atmospheric wavenumber spectra of wind and temperature observed by commercial aircraft. J. Atmos. Sci. 42, 950-960.