Using the full tensor of GOCE gravity gradients for regional gravity field modelling

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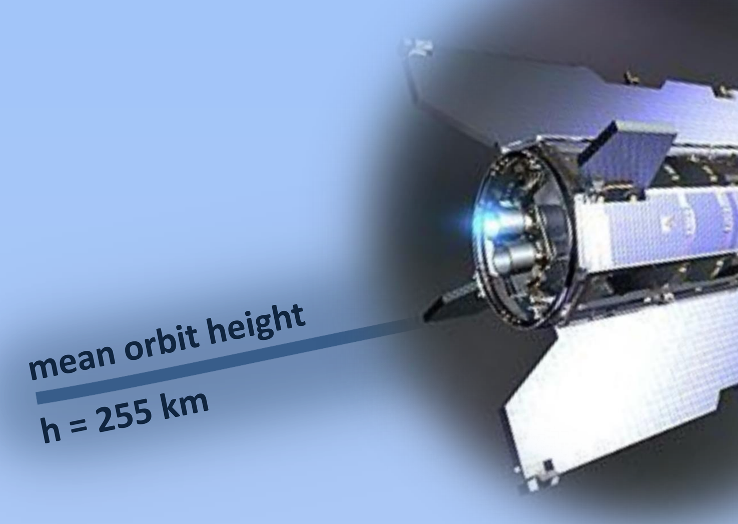
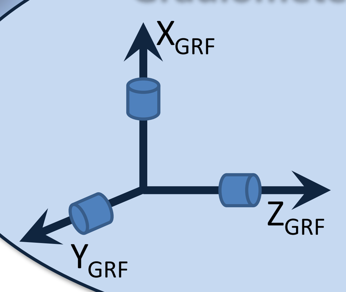


Gradiometer Reference Frame



O_{GRF} ... gradiometer centre of mass

X_{GRF} ≈ velocity direction

Y_{GRF} ≈ cross-track direction

Z_{GRF} ≈ radial direction

Measurement

- $V_{ab} = \left(\frac{\partial^2 V}{\partial a \partial b}\right) = \begin{bmatrix} V_{xx} & V_{xy} & V_{xz} \\ \ldots & V_{yy} & V_{yz} \\ \ldots & \ldots & V_{zz} \end{bmatrix}$ ... gravity gradients
- time span: 02/2010 – 06/2012

Pre-Processing

- filtering: cut-on frequency 5 mHz (degree $l \approx 27$) (highest sensitivity of GOCE within measurement band width MBW: 5 ... 100 mHz, related to an achievable spatial resolution up to $r \approx 80\ km$)
- filling up low frequencies with GOCO03S model
- subtracting background model V_{GOCO} : GOCO03S (d/o 250)

$\Delta V_{ab} = V_{ab} - V_{GOCO,ab}$

GOCE MBW

j [level]	1	2	3	4	5	6	7	8	9	10	11	12
l [deg]	1	3	7	15	31	63	127	255	511	1023	2047	4095
r [km]	20000	6667	2857	1333	645	317	157	78	39	20	10	5

frequency [deg] →

Abstract

With its 3-axis gradiometer GOCE delivers 3-dimensional (3D) information of the Earth's gravity field. The combination of all 6 GOCE gradients, observed in the Gradiometer Reference Frame (GRF), means an innovative challenge for regional gravity field modelling.

As the individual gravity gradients reflect the gravity field depending on different spatial directions, **observation equations** are formulated separately for each of these components. In our approach we use spherical localizing base functions to model the gravity field for specified regions (**analysis**). As output from the **synthesis** procedure we then obtain the second derivatives of the gravitational potential for all combinations of the xyz Cartesian coordinates in the Local North-Oriented Frame (LNOF).

Further the implementation of variance component estimation (VCE) provides a flexible tool to diversify the influence of the input gradiometer observations. Finally we compare the regional models with the static global GOCO03S model.

Summary

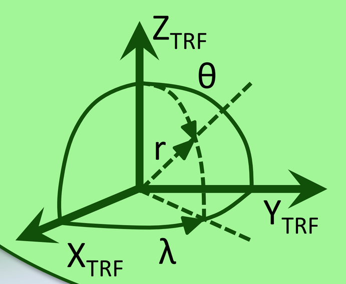
The GOCE gravity gradient grids obtained from different combinations of the xyz components show different structures of the Earth's disturbing potential and thus give information on the gravitational field depending on spatial directions. This essential advantage of the multidimensional measurement system can be used for research on the Earth's interior and for geophysical exploration.

Our regional approach further enables the consistent (spectral) combination with other gravity field observations which may provide more detailed structures for specified regions compared with global models. Therefore in the next steps,

- the comparison to a consistent filtered EGM2008 model,
- an entire error propagation and
- the optimization of the relative weighting and the filtering of the input data

have to be studied to analyse especially the signal content in the upper MBW of GOCE.

Terrestrial Reference Frame



O_{TRF} ... Earth's centre of mass

λ ... longitude

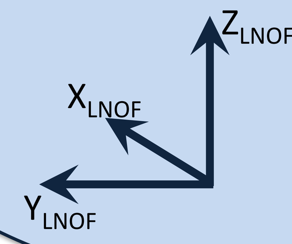
θ ... co-latitude

r ... radial distance

Earth's ellipsoid

WGS84 h = 0 km

Local North-Oriented Frame



O_{LNOF} ... satellite centre of mass

X_{LNOF} ... North

Y_{LNOF} ... West

Z_{LNOF} ... Up

Analysis

For each tensor element an observation equation is formulated (deterministic part)

- at level $J+1 = 8$ ($l = 255$),
- using reproducing kernels $\tilde{\phi}_{8,ab}$.

The unknown scaling coefficients d_7 are estimated by relative weighting of all observations using variance components (stochastic part).

Deterministic part

$$\Delta V_{ab}(x) + e_{ab}(x) = \tilde{\boldsymbol{\phi}}_{ab}{}^T(x)\, \boldsymbol{d}_7$$

IN: ΔV_{ab} observation

e_{ab} measurement error

$\tilde{\boldsymbol{\phi}}_{ab}$ (Nx1) vector of scaling functions

OUT: $\hat{\boldsymbol{d}}_7$ (Nx1) vector of scaling coefficients

Stochastic part

$$D(\boldsymbol{\Delta V_{ab}}) = \boldsymbol{\sigma_{ab}}^2 \boldsymbol{P_{ab}}^{-1}$$

IN: $\boldsymbol{\Delta V_{ab}}$ vector of observations

$\boldsymbol{P_{ab}}$ weighting matrix of observations (depending on data distribution)

OUT: $\hat{\boldsymbol{\sigma}}_{ab}$ variance components (VCs)

Rotation GRF - LNOF

Transformation LNOF - TRF

Observation equations

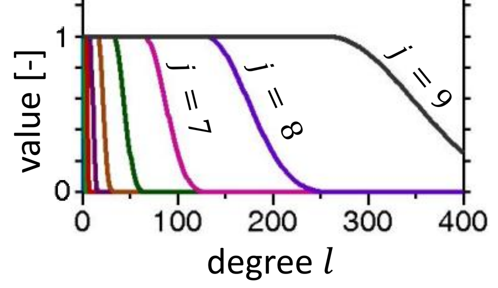
$$\Delta V_{ab} = \begin{bmatrix} \Delta V_{xx} & \Delta V_{xy} & \Delta V_{xz} \\ \ldots & \Delta V_{yy} & \Delta V_{yz} \\ \ldots & \ldots & \Delta V_{zz} \end{bmatrix} = \left(\sum_{q=1}^{N_J} \hat{\boldsymbol{d}}_{J,q}\ \tilde{\boldsymbol{\phi}}_{J+1,ab}(\boldsymbol{x}, \boldsymbol{x_q}) \right)$$

The reduced observations ΔV_{ab} can be described in series expansion using the estimated scaling coefficients $\hat{\boldsymbol{d}}_7$ (see **analysis**) and modified scaling functions with the components:

$$\tilde{\phi}_{8,ab} = \sum_{l=0}^{l_8'} \frac{2l+1}{4\pi} \Phi_{8,l} \left(\frac{R}{r}\right)^{l+1} \begin{cases} \frac{1}{r} \cdot P_l(\cos\psi)\left(-\frac{l+1}{r}\right) + \frac{1}{r^2} \cdot \frac{\partial P_l(\cos\psi)}{\partial\theta^2} & \cdots \Delta V_{xx} \\ \frac{1}{r^2 \sin\theta} \cdot \frac{\partial P_l(\cos\psi)}{\partial\lambda\, \partial\theta} - \frac{1}{r^2 \sin^2\theta} \cdot \frac{\partial P_l(\cos\psi)}{\partial\lambda} & \cdots \Delta V_{xy} \\ \frac{1}{r^2} \cdot \frac{\partial P_l(\cos\psi)}{\partial\theta} - \frac{1}{r} \cdot \left(-\frac{l+1}{r}\right) \cdot \frac{\partial P_l(\cos\psi)}{\partial\theta} & \cdots \Delta V_{xz} \\ \frac{1}{r} \cdot P_l(\cos\psi)\left(-\frac{l+1}{r}\right) + \frac{1}{r^2 \tan\theta} \cdot \frac{\partial P_l(\cos\psi)}{\partial\theta} + \frac{1}{r^2 \sin^2\theta} \cdot \frac{\partial P_l(\cos\psi)}{\partial\lambda^2} & \cdots \Delta V_{yy} \\ \frac{1}{r^2 \sin\theta} \cdot \frac{\partial P_l(\cos\psi)}{\partial\lambda} - \frac{1}{r \sin\theta} \cdot \left(-\frac{l+1}{r}\right) \cdot \frac{\partial P_l(\cos\psi)}{\partial\lambda} & \cdots \Delta V_{yz} \\ P_l(\cos\psi) \cdot \left(-\frac{l+1}{r}\right) & \cdots \Delta V_{zz} \end{cases}$$

Synthesis

frequency domain Blackman scaling fct. ϕ_j



IN: $\hat{\boldsymbol{d}}_7$ estimated coefficients

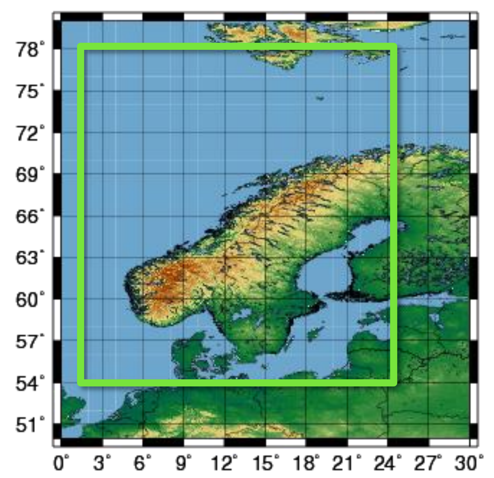
OUT: ΔV_{ab} gradients of the reduced gravitational potential

Modelling the reduced gravitational potential ΔV_{ab}

- at level $J+1 = 8$ ($l = 255$),
- using Blackman scaling functions $\tilde{\phi}_{8,ab}$,

as compromise between

- strongly band limited but declining function in frequency domain and
- oscillations in spatial domain.



North-East Atlantic margin

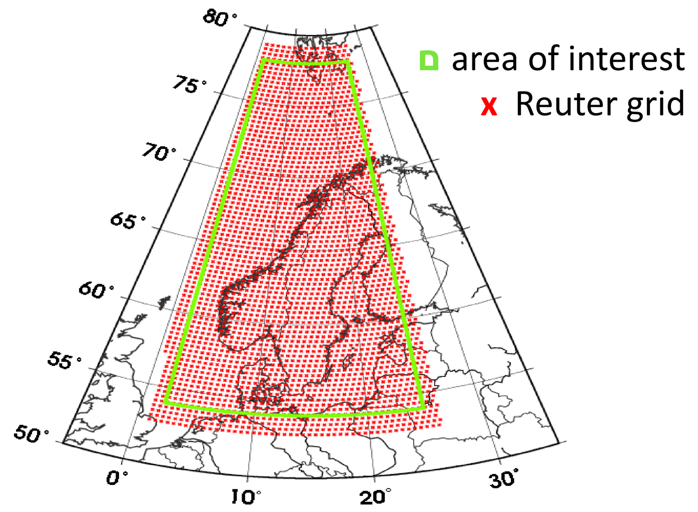
lon: 2 ... 25°, lat: 54 ... 78°

Modelling Approach

$$\Delta V(x) = \sum_{q=1}^{N} \sum_{l=0}^{l_J'} \frac{2l+1}{4\pi} d_{J,q} \phi_{J+1,l} \left(\frac{R}{r}\right)^{l+1} P_l(\cos\psi)$$

The reduced gravitational potential $\Delta V(\boldsymbol{x})$ can be expressed by series expansion in terms of scaling functions $\phi_{J+1,l}$.

- J max. resolution level (max. degree: l_J')
- $\phi_{J+1,l}$ scaling functions (located on a Reuter grid)
- $d_{J,q}$ scaling coefficients
- q number of grid points (max. number: N)
- R mean Earth radius
- $P_l(\cos\psi)$ Legendre polynomials
- ψ spherical distance angle between observation point (position vector $\boldsymbol{x}$) and computation point (position vector $\boldsymbol{x_q}$)



need

$$\begin{bmatrix} V_r \\ V_\lambda \\ V_\theta \end{bmatrix}, \begin{bmatrix} V_{rr} & V_{r\lambda} & V_{r\theta} \\ \ldots & V_{\lambda\lambda} & V_{\lambda\theta} \\ \ldots & \ldots & V_{\theta\theta} \end{bmatrix}$$

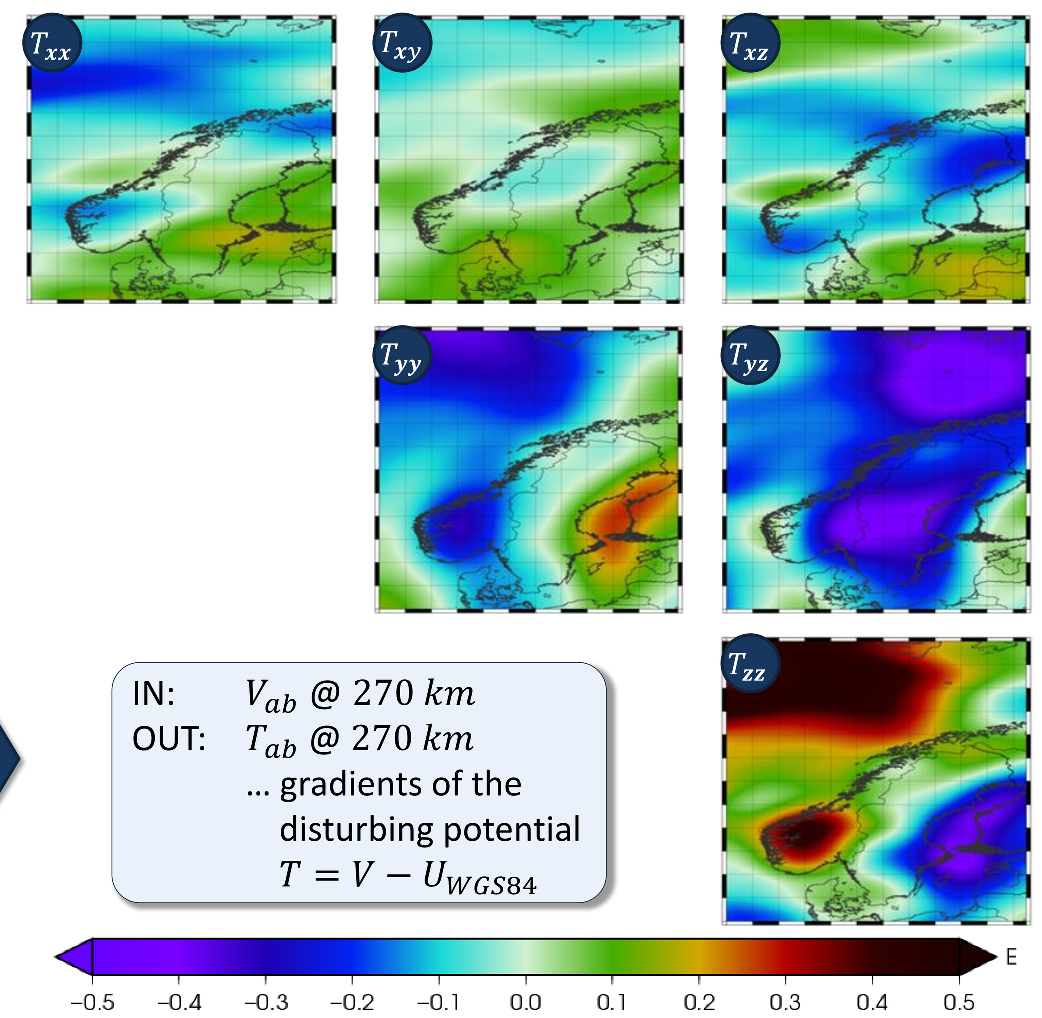
... 1st and 2nd derivatives of the residual potential $\Delta V(\boldsymbol{x})$ ... w.r.t. observation point $\boldsymbol{x}$

need

$$\begin{bmatrix} P_r \\ P_\lambda \\ P_\theta \end{bmatrix}, \begin{bmatrix} P_{rr} & P_{r\lambda} & P_{r\theta} \\ \ldots & P_{\lambda\lambda} & P_{\lambda\theta} \\ \ldots & \ldots & P_{\theta\theta} \end{bmatrix}$$

... 1st and 2nd derivatives of the Legendre polynomials $P_l(\cos\psi)$ ... w.r.t. spherical distance angle ψ

Results



IN: V_{ab} @ 270 km

OUT: T_{ab} @ 270 km ... gradients of the disturbing potential $T = V - U_{WGS84}$

Relative weighting of observations

- high VC $\hat{\boldsymbol{\sigma}}_{ab}$ → low weight $\frac{1}{\hat{\boldsymbol{\sigma}}_{ab}}$
- V_{GOCO} : prior information (reference, order of VC set to 1)

IN	order of VC	
	① est	② fix
V_{GOCO}	E+00	E+00
V_{xx}	E-02	E-02
V_{xy}	**E+00**	**E+11**
V_{xz}	E+01	E+00
V_{yy}	**E-02**	**E+03**
V_{yz}	**E+03**	**E+11**
V_{zz}	E-02	E-02

① est ... VC estimated

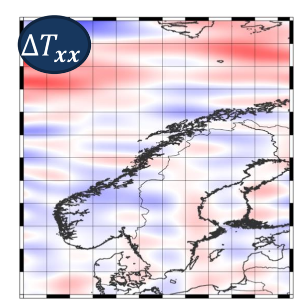
② fix ... VC manually set

The gravity gradient grids of the second derivatives of the disturbing potential show different structures depending on the spatial directions. The radial zz component shows the largest magnitude. The sum of the diagonal elements is approximately zero and thus fulfils the Laplace equation.

Down-weighting of the less accurate components V_{xy}, V_{yy} and V_{xy} might reduce the influence of systematic errors (smaller differences compared with GOCO03S).

Comparison with GOCO03S d/o 250

The differences in the xyz components vary between 0.04 mE ... 0.14 mE due to different signal content in the global model and the low-pass filtering of the observations in the regional approach.



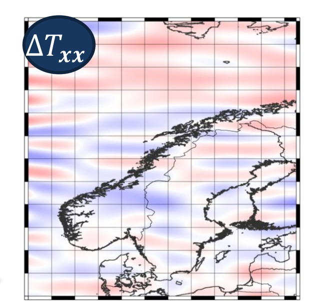
ΔT_{xx} @ 270 km

range: ±0.5 mE

std. dev. σ_{xx}

← ① 0.09 mE

② 0.06 mE →



Reference

Schmidt M., Fengler M., Mayer-Gürr T., Eicker A., Kusche J., Sanchez L., Han S.-C.: Regional Gravity Modelling in Terms of Spherical Base Functions. J Geod, 81, 17-38, doi: 10.1007/s00190-006-0101-5, 2007

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