

The Budyko framework beyond stationarity

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Introduction

Water availability is of major importance for a wide range of socio-economic sectors. Over land, the partitioning of precipitation (P) into evapotranspiration (E) and runoff (Q) is the key process to assess hydrological conditions. For climatological averages, the **Budyko framework** provides a simple first order relationship to estimate the evaporative index E/P as a function of aridity index (Ep/P, with Ep denoting potential evaporation). However, a major downside of the Budyko framework is its **limitation** to steady state conditions, being a result of the assumption of a closed land water balance. **Nonstationary** processes coming into play at other than mean annual catchment scales are thus not represented. Here we propose an analytically derived **new formulation of the Budyko curve** including an additional parameter being implicitly related to the nonlinear storage term of the land water balance.

Scope

- Deriving a **new formulation** of the **Budyko framework**
- Taking into account **nonstationarity** at shorter time scales
- **Evaluating** the framework against observations-based datasets

Motivation

The **original Budyko framework** is valid on **mean annual catchments scales only**.

Water balance is **closed**:
 $P - E - Q = 0$

On **shorter time scales** the **supply limit** is **violated** systematically.

Water balance is **not closed** as storage (S) changes play a role:
 $P - E - Q = dS/dt$

We derive a **new formula** implicitly accounting for storage changes by relaxing the supply limit

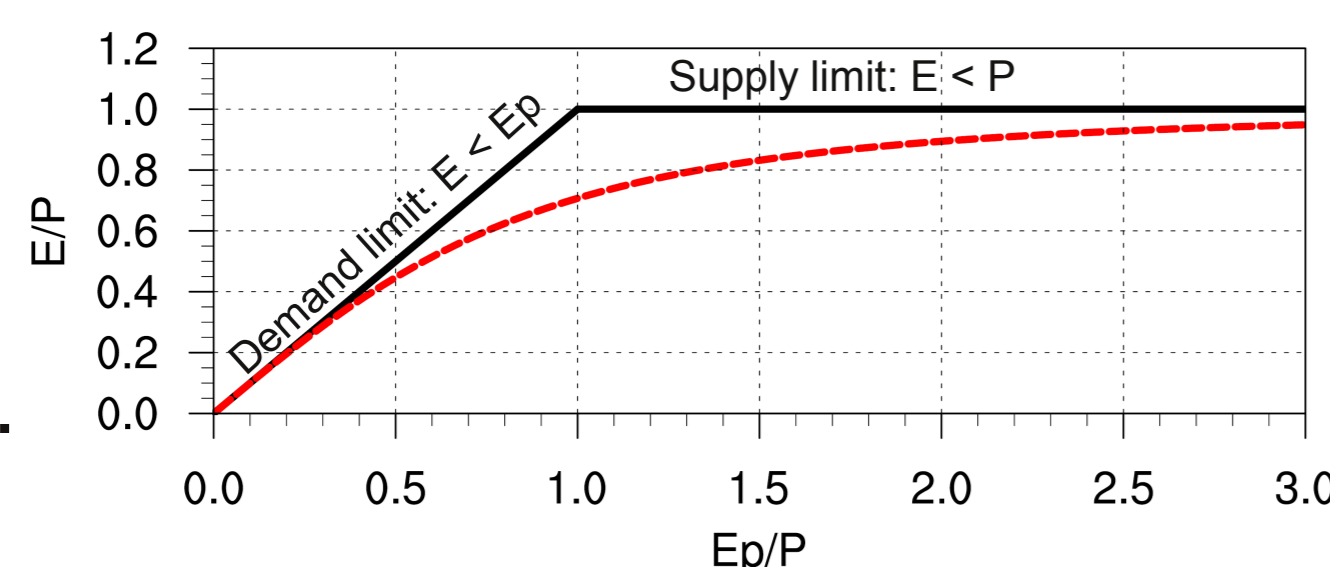


Fig. 1: The original Budyko framework. The black lines illustrate the physical limits. The red line shows the original Budyko curve.

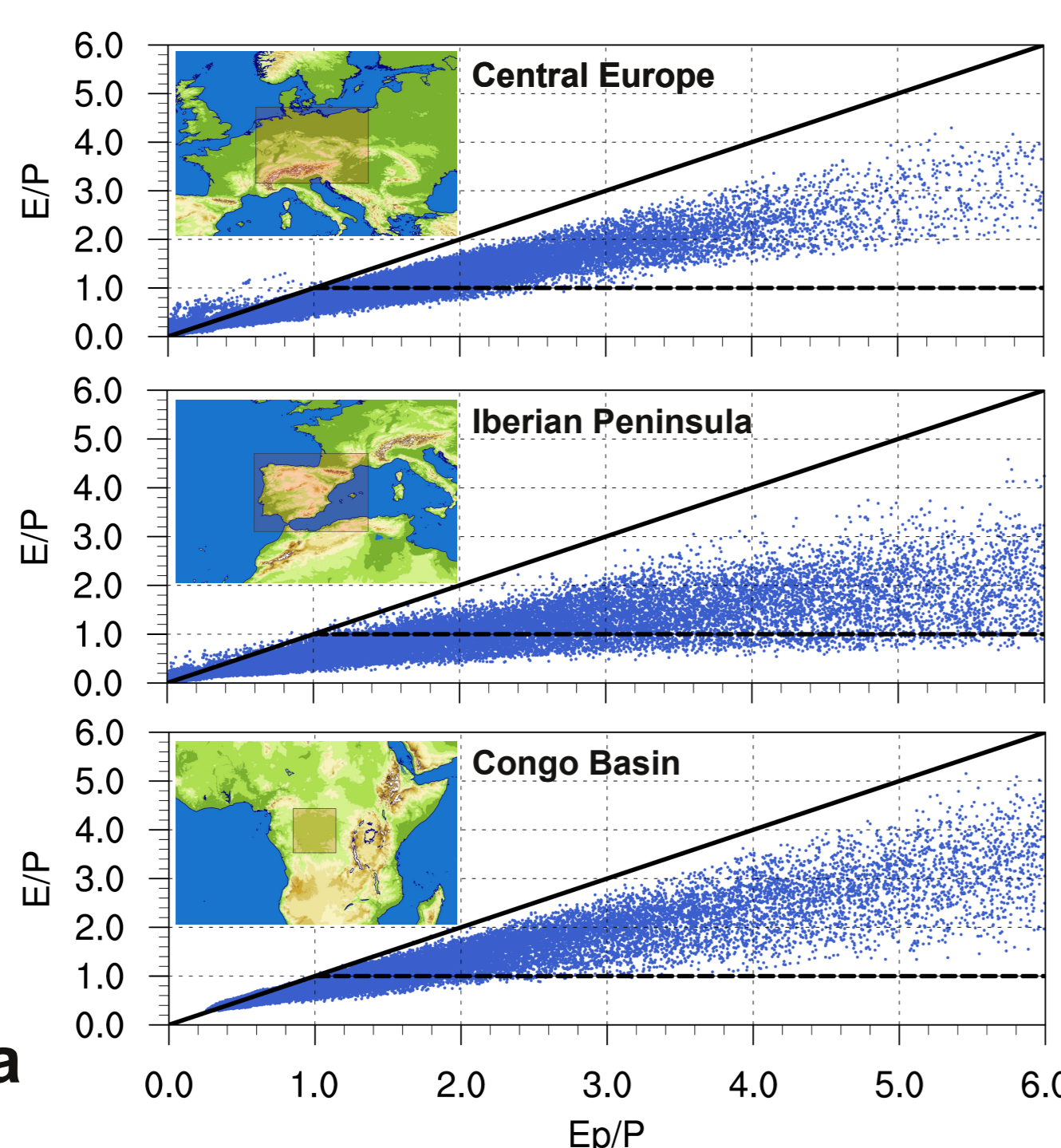


Fig. 2: Monthly values of Ep/P against E/P for all gridpoints within three spatial domains (top: Central Europe, middle: Iberia, bottom: Congo basin) representing areas of different climate conditions. The solid black line illustrates the demand limit, which is generally valid. The dashed black line represents the supply limit, being systematically violated.

Nonstationary Budyko

Fu, 1981 and Zhang et al., 2004 analytically derived a formula by defining the supply and demand limit as fixed **boundary conditions**. The obtained formula has the **free parameter ω** .
$$\frac{E}{P} = 1 + \left(\frac{E_p}{P}\right) - \left(1 + \left(\frac{E_p}{P}\right)^\omega\right)^{\frac{1}{\omega}}$$

Following their approach, but **relaxing the supply limit** implicitly accounts for the **nonlinear storage term**. Additionally, a second parameter is added.

$$\frac{E}{P} = 1 + \frac{E_p}{P} - \left(1 + (1 + y_0)^{\kappa-1} \left(\frac{E_p}{P}\right)^\kappa\right)^{\frac{1}{\kappa}}$$

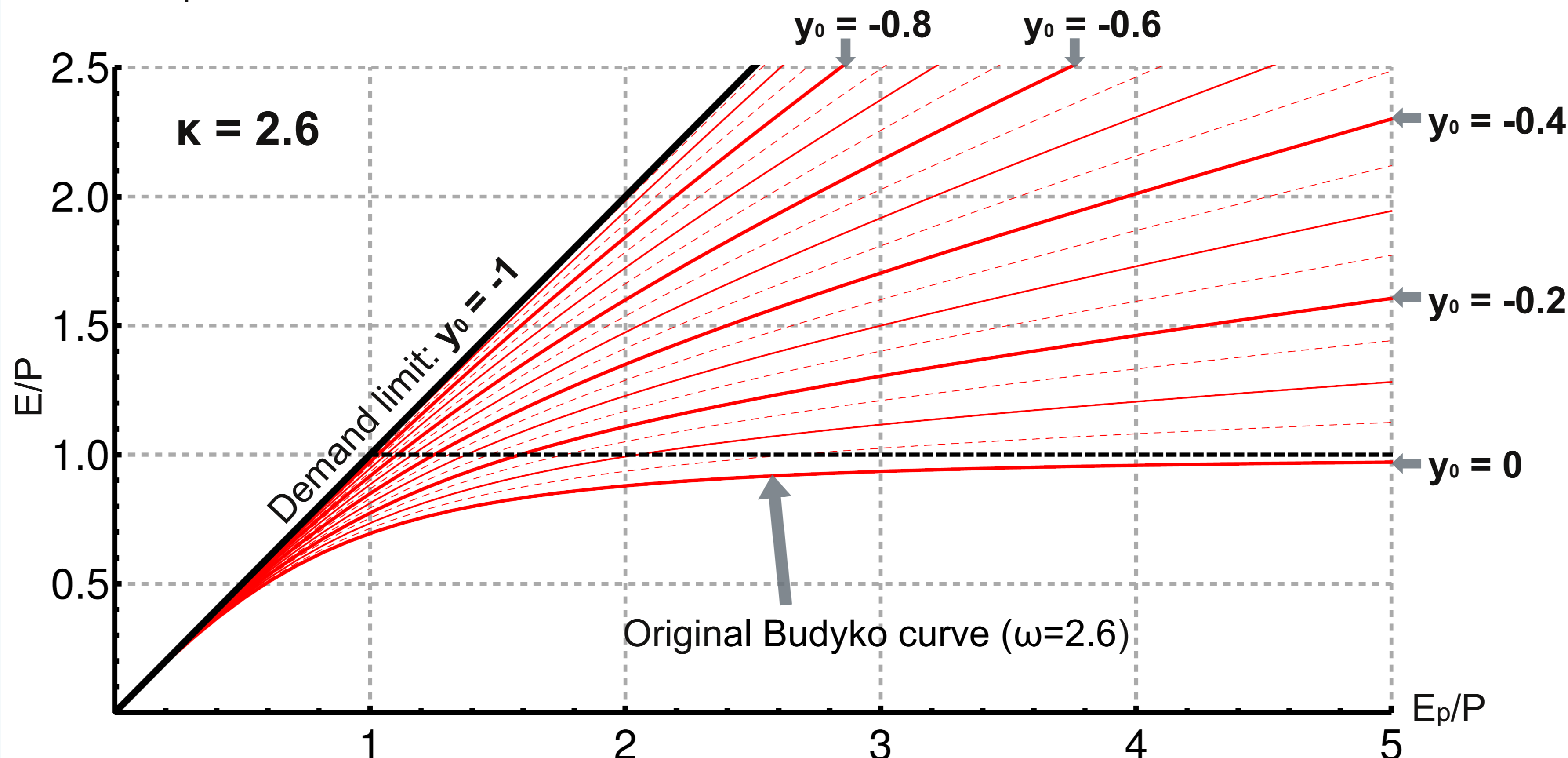


Fig. 3: Illustration of the obtained new framework for $\kappa=2.6$ and a large set of different y_0 (being the new second parameter). Note that the obtained curve for $(\kappa, y_0) = (2.6, 0)$ corresponds to the original Budyko curve ($\omega=2.6$). However, κ is not necessarily related to ω .

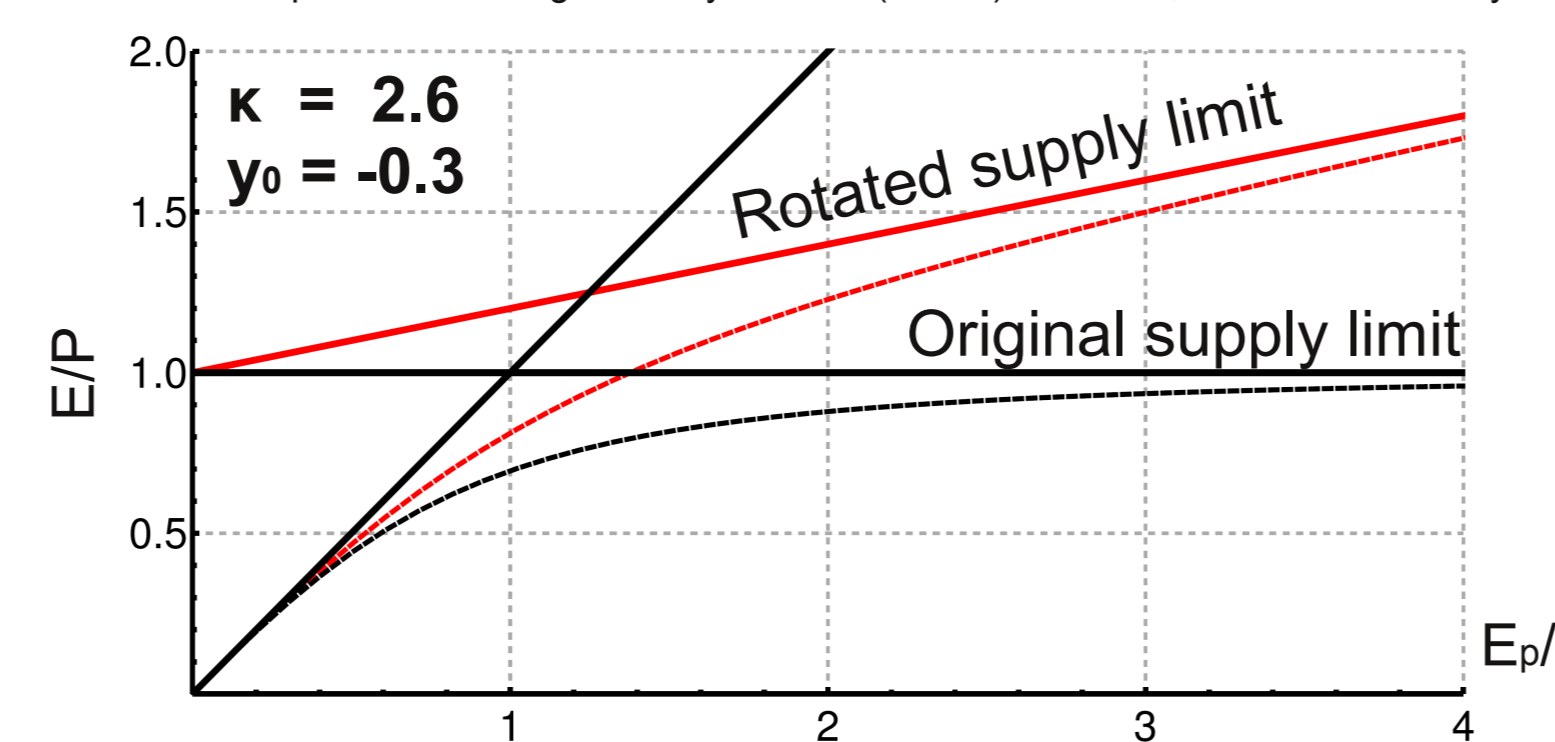


Fig. 4: The original supply limit at $E/P=1$ is rotated by the newly added parameter y_0 . The curve obtained from the new model (red, dashed) converges towards the new limit (red).

- The obtained **new curve** is described by a set of **two parameters** (κ, y_0).
- These parameters have no **a priori physical meaning** and are not necessarily related to ω .
- Technically, y_0 modifies the slope of the supply limit

Interpretation of ω , κ and y_0

The model parameters ω , κ and y_0 have no a priori physical meaning. While the interpretation of κ and y_0 needs to be investigated, the well established parameter ω is commonly interpreted as an integrator of catchment characteristics. Although, for ungauged catchments no estimation of ω is possible, previous studies have shown that ω is higher for forested than for grassland catchments (Zhang et al., 2004), related to NDVI for large catchments (Li et al., 2013) and influenced by various geolocation parameters (Xu et al., 2013).

More details

Leaflet with a detailed derivation of the new formula was attached here during the conference

All suggestions are welcome

Summary

- At other than mean annual catchment scales, the **supply limit** ($E/P < 1$) is **systematically violated**.
- Using **Fu's equation** we derived a flexible **two-parameter Budyko framework** by relaxing the supply limit.
- The new framework describes the phase space below the demand limit
- The new framework is capable to **estimate E** at **variable spatial and temporal scales**.
- Calibration and **evaluation** against observations-based datasets reveals an overall **good performance** of the new framework

References

- Adler, R. F., Huffman, G. J., Chang, A., Ferraro, R., Xie, P., Janowiak, J., Rudolf, B., Schneider, U., Curtis, S., Bolvin, D., Gruber, A., Susskind, J., Arkin, P., and Nelkin, E. (2003). The version-2 global precipitation climatology project (GPCP) monthly precipitation analysis (1979-present). *J. Hydrometeorol.*, 4:1147–1167.
- Budyko: Climate and life, Academic Press, June 1974
- Fu, B.P.: On the calculation of the evaporation from land surface (in Chinese), *Sci. Atmos. Sin.* 1(5), 23–31, 1981
- Li, Dan, Pan, Ming, Cong, Zhentao, Zhang, Lu, Wood, Eric: Vegetation control on water and energy balance within the Budyko framework, *Water Resources Research* 49(2), 969–976, 2013
- Mueller, B., Hirschi, M., Jimenez, C., Clais, P., Dirmeyer, P. A., Dolman, A. J., Fisher, J. B., Jung, M., Ludwig, F., Maignan, F., Miralles, D. G., McCabe, M. F., Reichstein, M., Sheffield, J., Wang, K., Wood, E. F., Zhang, Y., and Seneviratne, S. I.: Benchmark products for land evapotranspiration: LandFlux-EVAL multi-data set synthesis, *Hydrol. Earth Syst. Sci.*, 17, 3707–3720, 2013.13204.13226
- Sheffield, J., Wood, E., and Roderick, M. (2012). Little change in global drought over the past 60 years. *Nature*, 491:435–438.
- Xu, Xianli, Liu, Wen, Scanlon, Bridget R., Zhang, Lu, Pan, Ming: Local and global factors controlling water-energy balances within the Budyko framework, *Geophysical Research Letters* 40(23), 6123–6129, 2013
- Zhang, L., Hickel, K., Dawes, W. R., Chiew, F. H. S., Western, A. W., Briggs, P. R.: A rational function approach for estimating mean annual evapotranspiration, *Water Resources Research* 40(2), 2004

Model evaluation

- Pointwise **calibration** of (κ, y_0) against monthly gridded **observations-based datasets** for the 1990-2000 period.
- Generally **high correlation** with the E dataset used for calibration.
- Rather poor performance in tropical regions

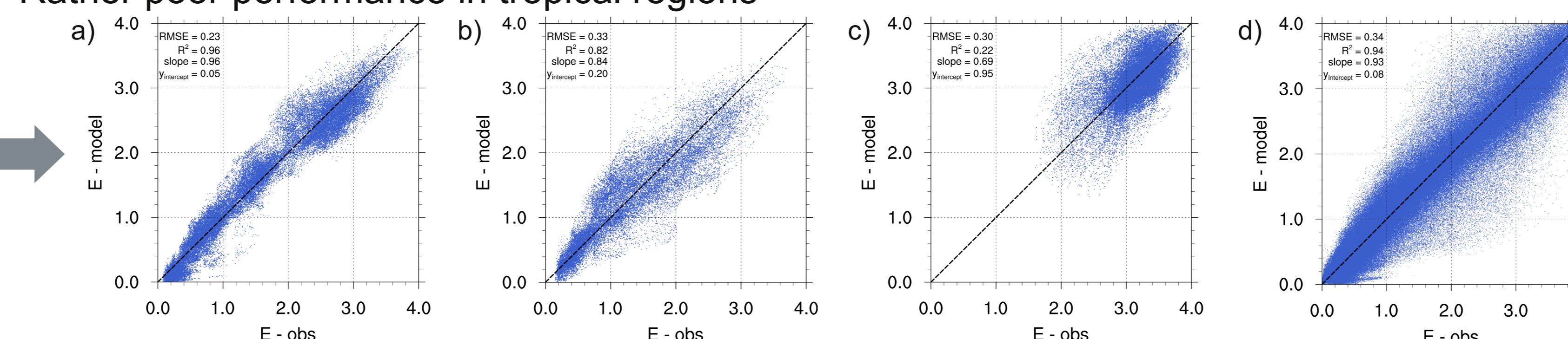


Fig. 5: Comparison of modeled E and observations-based E (Mueller et al., 2013) for a) Central Europe, b) Iberian Peninsula, c) Congo Basin (see also Fig. 2) and d) globally.

Datasets
For calibration and evaluation we use the LandFlux-Eval synthesis product (Mueller et al., 2013), being a comprehensive estimate of land E derived from various datasets. The choice of P and Ep datasets is almost arbitrary, as the obtained framework is calibrated against E. However, we use here the Global Precipitation Climatology Project (GPCP) dataset (Adler et al., 2003) and a pre-compiled Ep dataset (Sheffield et al., 2012) using the Penman-Monteith method.

