Replicate of the absolute magnetometer by K. F. Gauss.

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1 Summary

The aim of the experiment I describe below is to reproduce one of the two absolute intensity magnetometer developed by K. F. Gauss in the 1830's – for more details, I refer to Becquerel (1846) and Kono (2007). At this epoch, it was already known that a magnetized needle would oscillate in the presence of a magnetic field, and that the oscillation period was related to the field intensity. However, this period was only a relative measure of the magnetic field intensity, since the notion of magnetic moment was lacking. The principle is the following:

- in a first step, one measures the oscillation period T of a magnet $\mathcal{A}$ in the horizontal plane. We will see below in §2.1 that T is linked to the product $M_{\mathcal{A}}H$, with $M_{\mathcal{A}}$ the magnetic moment of the magnet and H the intensity of the horizontal component of the Earth's magnetic field;
- in a second step (section 2.2), one measures the deviation angle α^* of the magnet $\mathcal{A}$ (with respect to the geomagnetic North) in presence of a secondary magnet $\mathcal{B}$ (similar to $\mathcal{A}$) placed at a distance d. Approximating the field of $\mathcal{B}$ by a dipole, we relate $\alpha^*(d)$ to the ratio $M_{\mathcal{A}}/H$.

Knowing $M_{\mathcal{A}}H$ and $M_{\mathcal{A}}/H$, we deduce H with no prior knowledge on the magnetic moment of the magnets. I give some details below.

2 Theoretical background

2.1 Oscillation period of a magnetized needle

I write $\mathbf{M}_{\mathcal{A}}$ the (vector) magnetic moment of the magnet $\mathcal{A}$ (positionned in the horizontal plane), in the presence of $\mathbf{B}$ the vector magnetic field of the Earth. The magnetic torque, acting on $\mathcal{A}$, along the vertical unit vector $\mathbf{e}_z$ is

$$\Gamma = \mathbf{e}_z \cdot (\mathbf{M}_{\mathcal{A}} \times \mathbf{B}) = M_{\mathcal{A}} H \sin\alpha \,, \tag{1}$$

with H the horizontal component of the Earth's magnetic field and α the projection onto the horizontal plane of the angle between $\mathbf{M}_{\mathcal{A}}$ an $\mathbf{B}$. The equilibrium position is when $\mathcal{A}$ indicates the geomagnetic North. We know consider small deviation angles $\alpha \ll 1$ to the North. The conservation of angular momentum for the magnet $\mathcal{A}$ indicates

$$I_{\mathcal{A}} \frac{d\omega}{dt} = I_{\mathcal{A}} \frac{d^2\alpha}{dt^2} \simeq M_{\mathcal{A}} H \alpha \,, \tag{2}$$

with $\omega = d\alpha/dt$ the angular velocity and $I_{\mathcal{A}}$ the moment of inertia of the magnet. Solutions to eq. (2) are sines and cosines of period

$$T = 2\pi \sqrt{I_{\mathcal{A}}/(M_{\mathcal{A}} H)} \,. \tag{3}$$

2.2 Deviation angle of a magnetized needle

I now approximate the magnetic field $\mathbf{B}_{\mathcal{B}}$ of a magnet $\mathcal{B}$ (of magnetic moment $M_{\mathcal{B}}$), positionned in the horizontal plane, by a dipole – Gauss gave the framework for the dipole formulae when inventing spherical harmonics. The field intensity generated by the magnet $\mathcal{B}$, at a distance d of this magnet, is then

$$H_{\mathcal{B}}(d) \simeq \frac{\mu_0}{2\pi} \frac{M_{\mathcal{B}}}{d^3} \,, \tag{4}$$

where μ_0 is the magnetic permeability of free space. We consider the position of equilibrium for the magnet $\mathcal{A}$, sensitive too both $\mathbf{B}_{\mathcal{B}}$ and $\mathbf{B}$, with $H_{\mathcal{B}} \ll B$ (this last hypothesis is valid if $\mathcal{A}$ is not too clse to $\mathcal{B}$). It is such that $\mathbf{M}_{\mathcal{A}}$ is parrallel to $\mathbf{B} + \mathbf{B}_{\mathcal{B}}$. The magnet $\mathcal{A}$ is then deflected from the geomagnetic North by an angle $\alpha^* \ll 1$,

$$\alpha^* \simeq \sin \alpha^* = H_{\mathcal{B}}(d)/H \,, \tag{5}$$

where d is now the distance between the two magnets $\mathcal{A}$ and $\mathcal{B}$. Combining eq. (4) and (5) gives

$$\alpha^* \simeq \frac{\mu_0}{2\pi} \frac{M_{\mathcal{B}}}{H} d^{-3} \,. \tag{6}$$

If now we consider that the two magnets $\mathcal{A}$ and $\mathcal{B}$ are similar ($M_{\mathcal{B}} = M_{\mathcal{A}}$), we see that by measuring T and $\alpha^*(d)$, we can deduce H from eq. (3) and (6). Note that for the second step (estimation of the deflection angle α^*) one could replace $\mathcal{B}$ by $\mathcal{A}$, and take any magnet instead of $\mathcal{A}$.

Gauss performed this experiment in Göttingen in 1833. He estimated H by determining the two proportionality coefficients K_1 and K_2 for $\alpha^* = K_2 d^{-3}$ and $T^2 = K_1 I_{\mathcal{A}}$ (varying the moment of inertia $I_{\mathcal{A}}$ by moving little masses along the magnet $\mathcal{A}$). He found $H = 17.8\mu$T.

3 Replicating this instrument in practice

3.1 Dimensions for the experiment

moment of inertia

To get the dimensions of our system, we must first know the moment of inertia of the magnet. I will use to toy magnets 'geomag', that I consider as thin tall cylinders. I write ℓ their length, $S = \pi r^2$ their section (of radius $r = 3$ mm), and m their mass – implying a density $\rho = m/(\ell S)$. The moment of inertia is thus approximately

$$I_{\mathcal{A}} = \int_{-\ell/2}^{\ell/2} \rho S x^2 dx = \frac{m\ell^2}{12} \,. \tag{7}$$

With $m = 19.4$ g and $\ell = 11.6$ cm we find $I_{\mathcal{A}} \simeq 2.2\,10^{-5}$ kg.m^2.

magnetic moment

We also need its magnetic moment. We approximate it by that of a solenoid,

$$M_{\mathcal{A}} = \frac{B_{\mathcal{A}} S \ell}{\mu_0} \,, \tag{8}$$

with $B_{\mathcal{A}}$ the magnetic field inside the magnet. Taking $B_{\mathcal{A}} \simeq 0.5$ T, we find $M_{\mathcal{A}} \simeq 1.3$ A.m^2. This quantity is actually note required accurately to perform the experiment.

approximate period and deviation angle

We must check that the experiment can be run with a decent size, and a decent time... Given the values chosen above and using $H = 2\,10^{-5}$ T in eq. (3), I obtain $T \simeq 5$s, so that measuring T with O(10) periods should last of the order of 1 minute. For a distance between the two magnets of 50 cm (resp. 25 cm), the deviation angle is $\alpha \simeq 5°$ (resp. $\alpha \simeq 35°$): the experiment easily stands on a table.

3.2 Can we neglect the torsion torque of the wire?

Several precautions must be taken when running this experiment. First, the material for the frame should be nonmagnetic. The example shown below (Fig. 1) is built in duraluminium. Second, the torsional torque associated with the wire carying the magnet $\mathcal{A}$ should be negligeable, in comparison with the magnetic torque of eq. (1). For a full wire, it is given by

$$\Theta = G\theta I_0 \,. \tag{9}$$

Here $\theta = \alpha/L$ is the torsion angle per unit length. The wire, of length L, will be in nylon (actually a fishing line).

$$G = \frac{E}{2(1+\nu)} \tag{10}$$

is the shear modulus, with E the Young modulus (between 2.6 and 3 GPa for a nylon wire), and ν the Poisson coefficient ($\simeq 0.39$ for nylon).

$$I_0 = \frac{\pi \delta^4}{32} \tag{11}$$

is the moment of inertia for the full wire. Using a fishing line of diameter $\delta = 0.16$ mm and length $L = 50$ cm, one finds $\Theta = O(10^{-7})$ N.m (for $\alpha = 1$ rad). This should be compared with the magnetic torque. With the above magnet (of moment about 1 A.m^2) we have, in the presence of an ambient field $H = O(10^{-5})$ T, a magnetic torque $\Gamma \simeq 10^{-5}$ N.m (again for $\alpha = 1$ rad). We verify the hypothesis $\Gamma \gg \Theta$.

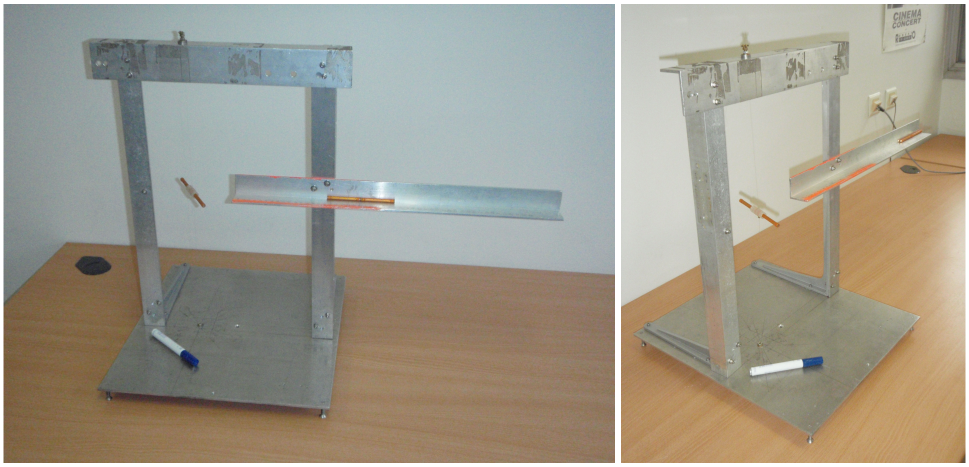

Figure 1: Photos of a prototype of the absolute intensity magnetometer, inspired by the experiment by K. F. Gauss.

References

- Becquerel A.C. (1846), *Traité complet du Magnétisme*. Eds. Firmin Didot frères.

- Kono M. (2007), *Geomagnetism in perspective*, in Treatise in Geophysics, Geomagnetism, vol 5, chap. 1, pp. 1-31, eds M. Kono & G. Schubert, Elsevier